A THEORETICAL INVESTIGATION OF THE M2 CONSTITUENT OF THE TIDE IN THE GULF OF MEXICO

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by

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TABLE OF SYMBOLS

a	the radius of the Earth
$A_1 \ldots A_n$	hypothetical coefficients in equations illustrating solution methods
$a_1, \ldots a_4$	coefficients in the iterative equations (see Appendix A for forms)
$b_1, \ldots b_4$	coefficients in the iterative equations (see Appendix A for forms)
c_1, c_2	constants in the equations in iterative form (see Appendix A for forms)
∇^2	$\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2}$
δ	declination of the theoretical body
d	distance from Earth to Moon
ϵ	grid distance
F	Coriolis parameter
φ	latitude
g	acceleration of gravity
H	amplitude of the forcing function
h	ocean depth below mean low water
K1	lunisolar diurnal tidal constituent
λ	relaxation coefficient
M2	principal lunar semidiurnal tidal constituent
M_m	mass of the Moon
M_e	mass of the Earth
N	number of terms in the equation
n	number of grid points in the grid
ψ	longitude relative to a given meridian, in this paper the reference is 90°W

R	Hansen's constant of proportionality for friction
σ	angular frequency of a given tidal constituent
u	velocity component in the X-direction (positive East) and independent of depth
uk_1	component of known velocity along the X-axis in phase with the forcing function
uk_2	component of known velocity along the X-axis out of phase with the forcing function
v	velocity component along the Y-axis (positive North) and independent of depth
vk_1	component of known velocity along the Y-axis in phase with the forcing function
vk_2	component of known velocity along the Y-axis out of phase with the forcing function
X	horizontal (in the beta-plane) coordinate axis oriented East-West and positive to the East
Y	horizontal (in the beta-plane) coordinate axis oriented North-South and positive to the North
ζ	tidal height above mean low water (positive upwards)
ζ_1	in-phase component of the tidal height
ζ_2	out-of-phase component of tidal height
$\bar{\zeta}$	forcing function

1. Introduction.

The objectives of this study are to develop a numerical method for estimating tidal constituents in the open ocean using fundamental theoretical concepts, to adapt the method to computer use, and to apply it to the analysis of the tidal regime in the Gulf of Mexico.

The ability to predict tidal behavior in the open sea is useful in oceanographic research and coastal engineering. It permits removal of tidal effects from deep-sea observations and prediction of the effect upon the tides of alterations of the coast line.

Present tidal knowledge is largely limited to the results of harmonic analysis at selected reference stations and empirical lag and amplitude corrections for closely adjacent stations. This technique is, of course, limited to the coast, but the work of Dietrich [1], subjectively fitting a two-constituent (M2 & K1) model (see below) to island and coastal date between coastlines extends the results into the open ocean. The accuracy of Dietrich's work is considered good in dense data regions, such as the North Atlantic; but in sparse data areas no general agreement has been reached concerning tidal regimes. The distortion of tides in shallow water cannot be accurately assessed. Thus the results of the method are, to some degree, in error.

A more satisfying approach is to apply the fundamental hydrodynamic equations and calculated luni-solar forcing function

to as realistic an ocean basin as possible.

Two methods of attack are the integration of the basic equations and the approximation of the equations by finite differences, leading to a solution by numerical methods. In either case, a boundary value problem results in which increased realism is accompanied by increased mathematical difficulty, whereas oversimplification can render the problem physically meaningless. In this paper discussion will be confined to numerical approaches. Hansen [2] used finite differences in his treatment of the M2 tidal constituent in the North Sea with excellent results, both in tidal heights and currents, as shown in Figure 1. Friction was included and the beta-plane approximation was used. Rossiter's approach [3] involves the use of spherical coordinates and frictionless flow in a meridionally bounded ocean of constant depth. Coriolis effects necessitate special care near the poles and the parameters were modified to insure convergence in these areas. In another application latitudinal boundaries were imposed to approximate the Atlantic Ocean. Relaxation techniques were employed successfully by both Hansen and Rossiter. Their results and Rossiter's recommendation that the method be applied to irregular shaped basins were important motivations for the present study.

2. Tides in the Gulf of Mexico

The tides in the Gulf of Mexico are modest in range, but diverse in character. Along the Gulf Coast the tides change abruptly and assume many forms. Figure 2 illustrates the distribution of these regimes. Along the entire Atlantic coast of the United States tides vary widely in range, but there is no variation in semi-diurnal character.

In the light of this constancy the abrupt changes in the Gulf are very dramatic. One example of considerable variation of the tidal regime is in the northeast portion of the Gulf. At Cedar Keys, a mixed, predominantly semi-diurnal regime exists. The tidal curves vary markedly in both directions along the coast. At Pensacola, 200 miles west of Cedar Keys, the diurnal influences dominate. Similar changes are encountered in other areas along the Gulf Coast. A comprehensive description of the tides along the Gulf Coast and the criteria for regime classification have been given by Marmer [4] . Many explanations have been advanced for these shifts in the basic form of the tides. Harris postulated in 1897 that the semi-diurnal tides were the extension of the Atlantic regime, while the diurnal tides resulted from the natural period of the Gulf basin.

Grace [5] compiled the various theories and applied an inventive analytical technique in 1932. He divided the Gulf into five sections in each of which the depth was treated as a

constant. The linearized, inviscid hydrodynamic equations of motion were solved within each section. Continuity was satisfied at the interior boundaries between sections and the boundary conditions of no normal flux were met at the coasts. The studies resulted in a unique set of corange and cotidal lines. Based on this work Grace put forth a theory of tidal dynamics in the Gulf: the semidiurnal tidal wave enters the Straits of Florida, moves around the Gulf coast in a contra solem amphidromy and exits through the Yucatan Strait six hours later; the diurnal tidal wave enters the Straits of Florida and propagates in such a way as to arrive at most points along the coast nearly simultaneously.

Several topographical features of the Gulf may well have discernable effects on the tides. The continental slope off the west coast of Florida is very steep and could act as a reflective surface to an incoming tidal wave. The intrusion of the Desoto Canyon toward the northern coast and the Mexican Basin are both relative deeps with unknown effects on the natural period of the Gulf. The Florida Keys and the broad shelf off the west coast of Florida could mask influences of tidal waves entering the Straits of Florida.

3. The Theoretical Development

Introduction

To describe the tides utilizing the basic hydrodynamic equations one must resort to describing the separate tidal constituents. This limits each solution to a particular known period and provides for a complete solution through a summation of constituents. Except in specialized cases this summation limits the problem to linearized equations of motion. The work described below is directed towards a solution for the principal lunar semidiurnal constituent, M2. Coastal observations indicate that knowledge of this constituent should permit explanation of these observations and description of regimes in the open sea. Harmonic analyses presented by Marmer indicate that M2 and K1, the lunisolar diurnal constituent, are the dominant constituents. The variability of the amplitude of K1 is from .59 feet to .09 feet, ignoring the results of short-term observations. The amplitude of M2 varies from zero to 1.07 feet by the same criterion. Thus the spacial distribution of the M2 constituent should be indicative of the distribution of the forms of the tides.

The Equation of Motion

From the equations of motion and continuity we may derive a second-order elliptic partial differential equation relating tidal height, depth, and their derivatives. The analysis is performed on a beta-plane. Advective and frictional terms in the equation of motion are neglected. The hydrostatic approximation is made.

The beta-plane assumption restricts the size of the basin in which the results will apply, but is considered acceptable in the Gulf.

Friction is present and does influence tidal distributions, especially in shallow water. Frictional effects are, however, believed to be highly non-linear, thus further complicating the equations. (The work of Dronkers[6] in linearizing this term for shallow water is an example of these complications.) Further justification for neglecting friction is that only the perimeter is in shallow water while most of the interior is in relatively deep water.

With the stated assumptions the dynamic equations are:

$$\begin{aligned} \frac{\partial u}{\partial t} - FV &= -g\frac{\partial(\zeta - \bar{\zeta})}{\partial x} \\ \frac{\partial v}{\partial t} + FU &= -g\frac{\partial(\zeta - \bar{\zeta})}{\partial y} \end{aligned} \qquad (1)$$

where,

u is the velocity component in the x direction (positive East),

v is the velocity component in the y direction (positive North),

F is the Coriolis parameter,

g is the acceleration of gravity,

ζ is the tidal height above undisturbed sea level (which is mean low water) and is positive upwards, and

$\bar{\zeta}$ is the forcing function (see below).

The hydrostatic approximation implies that u and v are independent of depth.

The equation of continuity is:

$$\frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} + \frac{\partial \zeta}{\partial t} = 0 \tag{2}$$

where h is the basin depth.

Since the frequency of the desired harmonic constituent is known, sinusoidal forms for the dependent variables are assumed:

$$\begin{aligned} \zeta &= Re\{\zeta(x,y)\, e^{i\sigma t}\} \\ u &= Re\{u(x,y)\, e^{i\sigma t}\} \\ v &= Re\{v(x,y)\, e^{i\sigma t}\} \\ \bar{\zeta} &= H \cos \sigma t \end{aligned} \tag{3}$$

where the notation, Re$\{\ \}$ indicates the real part of the quantity in brackets,

σ is the angular frequency of the constituent under examination, and

H is the amplitude of the forcing function.

Solving the equations of motion for u and v and substituting the results into the continuity equation yields one partial differential equation for the tidal height. Writing $\zeta = \zeta_1 + i\,\zeta_2$ (where, for instance, ζ_1 is the component in phase with the forcing function) and separating the differential equation into real and imaginary parts gives

$$h\nabla^2(\zeta_1 - H) + \frac{\partial h}{\partial x}\frac{\partial(\zeta_1 - H)}{\partial x} - \frac{F}{\sigma}\frac{\partial h}{\partial y}\frac{\partial \zeta_2}{\partial x} + \frac{\partial h}{\partial y}\frac{\partial(\zeta_1 - H)}{\partial y} + \frac{F}{\sigma}\frac{\partial h}{\partial x}\frac{\partial \zeta_2}{\partial y} + \frac{\sigma^2 - F^2}{g}\zeta_1 = 0$$

and

$$h\nabla^2\zeta_2 + \frac{\partial h}{\partial x}\frac{\partial \zeta_2}{\partial x} + \frac{F}{\sigma}\frac{\partial h}{\partial y}\frac{\partial(\zeta_1 - H)}{\partial x} + \frac{\partial h}{\partial y}\frac{\partial \zeta_2}{\partial y} - \frac{F}{\sigma}\frac{\partial h}{\partial x}\frac{\partial(\zeta_1 - H)}{\partial y} + \frac{\sigma^2 - F^2}{g}\zeta_2 = 0 \tag{4}$$

where $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$

Rossiter used spherical coordinates in treating basins covering large areas of the earth. Thus his equations differ in detail at this point although the form is very similar.

Hansen's work in the North Sea included friction. The added terms in the equations of motion had the form,

$$F_x = Ru \quad \text{and} \quad F_Y = RV \qquad (5)$$

where R is a suitably chosen proportionality factor. The excellence of his results is a strong argument for this somewhat ad hoc hypothesis. In the situation involving considerably greater bottom relief this form is inadequate due to the variability of the friction effect with depth. In deep water friction will not influence tidal distributions as much as bottom topography which enters the equations through continuity considerations.

Boundary Conditions

The boundary conditions determine the velocities normal to the boundaries. At solid boundaries these velocities must be zero. At the passages into the Gulf values must be assigned to the associated tidal currents. Following the treatment of tidal heights, we write

$$\begin{aligned} U &= UK1 + i\,UK2 \\ V &= VK1 + i\,VK2. \end{aligned} \qquad (6)$$

Substituting these relations into the solutions for u and v from the equations of motion and separating into real and imaginary parts gives

$$VK1 = -\frac{g}{2\epsilon(\sigma^2-F^2)}\left\{\sigma\frac{\partial\zeta_2}{\partial y} + F\frac{\partial(\zeta_1-H)}{\partial x}\right\}$$

$$VK2 = \frac{g}{2\epsilon(\sigma^2-F^2)}\left\{\sigma\frac{\partial(\zeta_1-H)}{\partial y} - F\frac{\partial}{\partial x}\zeta_2\right\}$$

$$UK1 = -\frac{g}{2\epsilon(\sigma^2-F^2)}\left\{\sigma\frac{\partial\zeta_2}{\partial x} - F\frac{\partial(\zeta_1-H)}{\partial y}\right\} \quad (7)$$

$$UK2 = \frac{g}{2\epsilon(\sigma^2-F^2)}\left\{\sigma\frac{\partial(\zeta_1-H)}{\partial x} + F\frac{\partial\zeta_2}{\partial y}\right\}.$$

Observations of tidal currents to determine their constituents are not frequently made. For accuracy continuous observations are required over long periods of time. Tidal current measurements made by Harris in 1887 and supported by Grace's work were used in the Yucatan and Florida Straits in this study. The accuracy of these calculations is questionable but the order of magnitude is correct. By allowing flow through these channels the effects of a co-oscillating tide are included in the solution.

The Forcing Function

The forcing function is basically the gravitational effect of the ideal body associated with the constituent to be examined. It is obtained from equilibrium tidal theory (for example see Proudman[7]) in the form,

$$H = \alpha\left(\frac{a}{d}\right)^3 \frac{M_m}{M_e}\left\{\frac{3}{4}\cos^2\varphi\cos^2\delta\cos 2\psi\right\} \quad (8)$$

for the lunar semidiurnal constituents, and

$$H = a\left(\frac{a}{d}\right)^3 \frac{Mm}{Me}\left\{\frac{3}{4}\sin 2\varphi \cos 2\delta \cos \psi\right\} \qquad (9)$$

for the lunar diurnal constituents.

Here,

a is the radius of the Earth

d is the distance from the Earth to the Moon

Mm is the mass of the Moon

Me is the mass of the Earth

φ is the latitude of the point under consideration

δ is the declination of the ideal body

ψ is the longitude relative to the reference longitude.

For computations the central meridian of the Gulf, 90°W, will be used as a reference.

Similar expressions apply for solar diurnal and semidiurnal constituents and also for those with periods of up to two weeks. In this work these constituents have relatively small amplitudes and so are not treated.

Over the area of the Gulf the lunar semidiurnal forcing function varies by about eighteen percent.

4. Numerical Techniques

Finite Difference Approximations

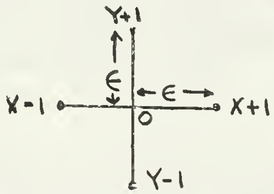
To put equations (4) into a form more suitable for numerical treatment the derivatives are approximated by central differences,

$$\left(\frac{\partial z}{\partial X}\right)_0 = \frac{z_{X+1} - z_{X-1}}{2\epsilon} \quad , \quad \left(\frac{\partial z}{\partial Y}\right)_0 = \frac{z_{Y+1} - z_{Y-1}}{2\epsilon} \qquad (10)$$

and

$$\left(\nabla^2 z\right)_0 = \frac{1}{\epsilon^2}\left\{z_{X+1} + z_{X-1} + z_{Y+1} + z_{Y-1} - 4 z_0\right\} \qquad (11)$$

where z is a dummy variable and the subscripts refer to location in the grid arrangement as shown below.



Central differences utilize values sampled at an interval of two grid distances. This has a smoothing effect on the data. Because the bathymetry is normally given to the nearest fathom smoothing is beneficial in decreasing errors in depth gradient due to sounding inaccuracies. The tidal heights are much smaller than the depths. Because of this, solutions are very sensitive to sounding errors. For this reason central differences are used in all finite difference approximations.

The use of central differences at a point on the boundary introduces values for tidal heights outside the boundary into the equations.

For an interior point, equations (4) with finite differences become

$$(\zeta_1 - H)_{X+1}(4h_0 + h_{X+1} - h_{X-1}) + (\zeta_1 - H)_{X-1}(4h_0 - h_{X+1} + h_{X-1})$$
$$+ (\zeta_1 - H)_{Y+1}(4h_0 + h_{Y+1} - h_{Y-1}) + (\zeta_1 - H)_{Y-1}(4h_0 - h_{Y+1} + h_{Y-1})$$
$$+ (\zeta_1 - H)_0(-16h_0) + (\zeta_2)_{X+1}\left[-\frac{F}{\sigma}(h_{Y+1} - h_{Y-1})\right]$$
$$+ (\zeta_2)_{X-1}\left[\frac{F}{\sigma}(h_{Y+1} - h_{Y-1})\right] + (\zeta_2)_{Y+1}\left[\frac{F}{\sigma}(h_{X+1} - h_{X-1})\right]$$
$$+ (\zeta_2)_{Y-1}\left[-\frac{F}{\sigma}(h_{X+1} - h_{X-1})\right] + (\zeta_1)_0\left[4\epsilon^2\frac{(\sigma^2 - F^2)}{g}\right] = 0$$

and (12)

$$(\zeta_2)_{X+1}\left[4h_0 + h_{X+1} - h_{X-1}\right] + (\zeta_2)_{X-1}(4h_0 - h_{X+1} + h_{X-1})$$
$$+ (\zeta_2)_{Y+1}(4h_0 + h_{Y+1} - h_{Y-1}) + (\zeta_2)_{Y-1}(4h_0 - h_{Y+1} + h_{Y-1})$$
$$+ (\zeta_2)_0(-16h_0) + (\zeta_1 - H)_{X+1}\left[+\frac{F}{\sigma}(h_{Y+1} - h_{Y-1})\right]$$
$$+ (\zeta_1 - H)_{X-1}\left[-\frac{F}{\sigma}(h_{Y+1} - h_{Y-1})\right] + (\zeta_1 - H)_{Y+1}\left[-\frac{F}{\sigma}(h_{X+1} - h_{X-1})\right]$$
$$+ (\zeta_1 - H)_{Y-1}\left[\frac{F}{\sigma}(h_{X+1} - h_{X-1})\right] + (\zeta_2)_0\left[4\epsilon^2\frac{(\sigma^2 - F^2)}{g}\right] = 0.$$

Consideration was given to variable grid distancing to permit detailed analysis in areas of large depth gradients. However, the stability of the computation about a point with different distances to each of the adjacent points is questionable, and the finite difference form of the equations is quite complex. Appendix B contains the finite difference equations for a variable grid distance. The density of bathymetric data in the Gulf is such that a grid finer than that used below would

require considerable interpolation and thus no improvement in the solution would appear.

Rossiter used a finite difference equation involving derivatives at the boundary and three interior points to produce the tidal heights outside the boundary. Although it is valid, Rossiter's procedure has the disadvantage of introducing parameters associated with non-adjacent points into the equations. This complicates the system of equations in a relaxation technique by increasing the number of points involved in each equation by two. It also disrupts the iterative form designed for computer use, which will be discussed later.

At a boundary point, the basic equations (12), derived from continuity considerations and the applicable boundary conditions are both in effect. These boundary conditions manifest themselves in the forms of prescribed velocities normal to the boundaries.

The velocity equations (7) in finite difference form are

$$VK1 = \frac{-g}{2\epsilon(\sigma^2 - F^2)} \left\{ \sigma \left[(\zeta_2)_{Y+1} - (\zeta_2)_{Y-1} \right] + F \left[(\zeta_1 - H)_{X+1} - (\zeta_1 - H)_{X-1} \right] \right\}$$

$$VK2 = \frac{g}{2\epsilon(\sigma^2 - F^2)} \left\{ \sigma \left[(\zeta_1 - H)_{Y+1} - (\zeta_1 - H)_{Y-1} \right] - F \left[(\zeta_2)_{X+1} - (\zeta_2)_{X-1} \right] \right\}$$

$$(13)$$

$$UK1 = \frac{-g}{2\epsilon(\sigma^2 - F^2)} \left\{ \sigma \left[(\zeta_2)_{X+1} - (\zeta_2)_{X-1} \right] - F \left[(\zeta_1 - H)_{Y+1} - (\zeta_1 - H)_{Y-1} \right] \right\}$$

$$UK2 = \frac{g}{2\epsilon(\sigma^2 - F^2)} \left\{ \sigma \left[(\zeta_1 - H)_{X+1} - (\zeta_1 - H)_{X-1} \right] + F \left[(\zeta_2)_{Y+1} - (\zeta_2) \right] \right\}.$$

Thus, for a given normal velocity, the tidal height for the point outside the boundary may be solved for in terms of three interior tidal heights. Substituting these solutions into the equation for an interior point now incorporates the dynamic equations, the continuity equation and the boundary condition into a set of two equations at the point, as well as eliminating values outside the boundary from the calculations.

The substitutions and resultant equations will be unique for each boundary or corner. As an example, to solve for a point on the south boundary with a known velocity, equations (13) are used in the form,

$$(\zeta_1 - H)_{Y-1} = (\zeta_1 - H)_{Y+1} - \frac{F}{\sigma}(\zeta_2)_{X+1} + \frac{F}{\sigma}(\zeta_2)_{X-1} - VK2\left(\frac{2\epsilon(\sigma^2 - F^2)}{g\sigma}\right) \tag{14}$$

$$(\zeta_2)_{Y-1} = (\zeta_2)_{Y+1} + \frac{F}{\sigma}(\zeta_1 - H)_{X+1} - \frac{F}{\sigma}(\zeta_1 - H)_{X-1} + VK1\left(\frac{2\epsilon(\sigma^2 - F^2)}{g\sigma}\right)$$

and substituted into the interior equations (12) to eliminate values involving the exterior point Y-1.

This results in

$$\begin{aligned}
&(\zeta_1 - H)_{Y+1}\left[8h_0\right] + (\zeta_2)_{X+1}\left[\frac{F}{\sigma}4h_0\right] + (\zeta_2)_{X-1}\left[\frac{F}{\sigma}4h_0\right] + \\
&+ (\zeta_1 - H)_{X+1}\left[4h_0 + (h_{x+1} - h_{x-1})\left(1 - \frac{F^2}{\sigma^2}\right)\right] \\
&+ (\zeta_1 - H)_{X-1}\left[4h_0 - (h_{x+1} - h_{x-1})\left(1 - F^2/\sigma^2\right)\right] + 4h_0 H \qquad (15) \\
&+ (\zeta_1)_0\left[\frac{4\epsilon^2(\sigma^2 - F^2)}{g} - 16h_0\right] - VK1\left[\frac{2\epsilon(\sigma^2 - F^2)}{g\sigma^2}F(h_{x+1} - h_{x-1})\right] \\
&- VK2\left[\frac{2\epsilon(\sigma^2 - F^2)}{g}(4h_0 - h_{y+1} + h_{y-1})\right] = 0
\end{aligned}$$

and

$$
\begin{aligned}
&(\zeta_2)_{Y+1}\left[8h_0\right] + (\zeta_1 - H)_{X+1}\left[\tfrac{F}{\sigma}4h_0\right] + (\zeta_1 - H)_{X-1}\left[4\tfrac{F}{\sigma}h_0\right] \\
&+ (\zeta_2)_{X+1}\left[4h_0 + (h_{X+1} - h_{X-1})(1 - F^2/\sigma^2)\right] \\
&+ (\zeta_2)_{X-1}\left[4h_0 - (h_{X+1} - h_{X-1})(1 - F^2/\sigma^2)\right] \\
&+ (\zeta_2)_0\left[\frac{4\epsilon^2(\sigma^2 - F^2)}{g} - 16h_0\right] \\
&+ VK1\left[\frac{2\epsilon(\sigma^2 - F^2)}{g\sigma}(4h_0 - h_{Y+1} + h_{Y-1})\right] \\
&- VK2\left[\frac{2\epsilon(\sigma^2 - F^2)F}{g\sigma^2}(h_{X+1} - h_{X-1})\right] = 0.
\end{aligned}
$$

To accommodate the boundary conditions involved at a corner values of u and v must enter the basic equations (12). This is accomplished by simultaneous solution of equations (13) to eliminate terms involving the two heights at the same point from the combined equations. The results are

$$
\begin{aligned}
(\zeta_2)_{Y+1} &= (\zeta_2)_{Y-1} - UK2\left[\frac{2\epsilon F(\sigma^2 - F^2)}{g\sigma^2(1 - F^2/\sigma^2)}\right] - VK1\left[\frac{2\epsilon(\sigma^2 - F^2)}{g\sigma(1 - F^2/\sigma^2)}\right] \\
(\zeta_1 - H)_{Y+1} &= (\zeta_1 - H)_{Y-1} - UK1\left[\frac{2\epsilon F(\sigma^2 - F^2)}{g\sigma^2(1 - F^2/\sigma^2)}\right] + VK2\left[\frac{2\epsilon(\sigma^2 - F^2)}{g\sigma(1 - F^2/\sigma^2)}\right] \\
(\zeta_2)_{X+1} &= (\zeta_2)_{X-1} + VK2\left[\frac{2\epsilon F(\sigma^2 - F^2)}{g\sigma^2(1 - F^2/\sigma^2)}\right] - UK1\left[\frac{2\epsilon(\sigma^2 - F^2)}{g\sigma(1 - F^2/\sigma^2)}\right] \qquad (16) \\
(\zeta_1 - H)_{X+1} &= (\zeta_1 - H)_{X-1} + VK1\left[\frac{2\epsilon F(\sigma^2 - F^2)}{g\sigma^2(1 - F^2/\sigma^2)}\right] + UK2\left[\frac{2\epsilon(\sigma^2 - F^2)}{g\sigma(1 - F^2/\sigma^2)}\right].
\end{aligned}
$$

Combinations of these equations will meet any corner boundary condition. For example, substituting to eliminate values at the

point X+1 in the south boundary equations will give equations for the southeast corner in the form,

$$UK_1\left[\frac{4\,F/\sigma\, h_0(\sigma^2-F^2)\,2\epsilon}{g\sigma\,[1-F^2/\sigma^2]}\right] + (\zeta_1-H)_{X-1}\,[8h_0]$$
$$+ (\zeta_1-H)_{Y+1}\,[8h_0] + VK1\left[\frac{8h_0 F\epsilon(\sigma^2-F^2)}{g\sigma^2(1-F^2/\sigma^2)}\right]$$
$$- UK2\left[\left(\frac{2\epsilon(\sigma^2-F^2)}{g\sigma}\right)\left(\frac{4h_0}{1-F^2/\sigma^2}+h_{X+1}-h_{X-1}\right)\right]$$
$$- VK2\left[\left(\frac{2\epsilon(\sigma^2-F^2)}{g\sigma}\right)\left(4h_0-h_{Y+1}+h_{Y-1}+\frac{4F^2h_0}{\sigma^2(1-F^2/\sigma^2)}\right)\right]$$
$$+\frac{4H\epsilon^2(\sigma^2-F^2)}{g} + (\zeta_1-H)_0\left\{\frac{4\epsilon^2(\sigma^2-F^2)}{g}-16h_0\right\}$$
$$= 0$$

(17)

and

$$UK2\left[\frac{8\,F/\sigma\, h_0\,\epsilon(\sigma^2-F^2)}{g\sigma(1-F^2/\sigma^2)}\right] + (\zeta_2)_{X-1}\,[8h_0]$$
$$+ (\zeta_2)_{Y+1}\,[8h_0] + VK2\left[\frac{8Fh_0\,\epsilon(\sigma^2-F^2)}{g\sigma^2(1-F^2/\sigma^2)}\right]$$
$$+ UK1\left[\left(\frac{2\epsilon(\sigma^2-F^2)}{g\sigma}\right)\left(\frac{4h_0}{(1-F^2/\sigma^2)}+h_{X+1}-h_{X-1}\right)\right]$$
$$+ VK1\left[\left(\frac{2\epsilon(\sigma^2-F^2)}{g\sigma}\right)\left(4h_0-h_{Y+1}+h_{Y-1}+\frac{4F^2h_0}{\sigma^2(1-F^2/\sigma^2)}\right)\right]$$
$$+ (\zeta_2)_0\left[\frac{4\epsilon^2(\sigma^2-F^2)}{g}-16h_0\right] = 0.$$

The forms of the equations for each boundary and corner point are presented in Appendix A.

Methods of Solution

When equations (12) and their equivalents incorporating boundary conditions (see Appendix A) are applied at each point of a grid consisting of n points describing the body of water studied, a system of 2n equations in 2n unknown tidal heights results. Both methods of solution undertaken here are predicated upon the presence of a dominant diagonal in the system's coefficient matrix. That is, in the application of the equations to each point there is one coefficient much larger than all others in the equation and in each successive equation the largest coefficient multiplies a different unknown value of the tidal height. Both methods are designed to obtain a solution by altering the unknown tidal height associated with the largest coefficient.

Gauss - Seidel iteration (see for example, Hildebrand [8]) is the method used in the computer program. Briefly, this method consists of obtaining new values for the variable with the dominant coefficient, say X_1 , in an equation of the form,

$$A_1 X_1 + A_2 X_2 + \cdots + A_N X_N = 0 \qquad (17)$$

by the formula

$$X_1 = -\frac{1}{A_1} \sum_2^N A_N X_N . \qquad (18)$$

This satisfies the equation with approximate values of $X_1, \ldots,$ X_n by varying X_1 only. This procedure is carried out for each equation in the system following a first guess of the value of

each variable. Repeated use of equation (18) usually brings about "convergence", in the sense that each of the equations is satisfied by the same set of unknowns to within prescribed tolerances. Gauss - Seidel iteration is not highly sensitive to the first guess values.

Relaxation is a similar process but permits selective variation of the unknowns to bring about convergence. Using the same example equation the first guess will give

$$A_1X_1 + A_2X_2 + \cdots + A_nX_n = R$$

where R is the "residual", which should vanish to achieve convergence of the system of equations. Changes will be made in the unknown associated with the largest coefficient following the rule

$$X_1' = X_1 - \lambda \frac{R}{A_1} \qquad (19)$$

where λ is the "Relaxation Coefficient" and may take on values in the range

$$0 < \lambda < 2. \qquad (20)$$

When $\lambda = 1$ the residual will be reduced to zero and the process is very similar to the iteration described above. When $\lambda > 1$ the residual tends to alternate sign on successive iterations and decrease in magnitude. This process usually speeds up convergence and is called overrelaxation. When $\lambda < 1$ the residual tends to be reduced in magnitude and keep the same sign thus approaching zero from one side rather than oscillating around zero. This is called underrelaxation.

5. Hand Calculation of the M2 Tidal Constituent in the Gulf

To gain an understanding of the behavior of a tidal solution in the Gulf and to supplement the computer work a hand solution was undertaken for a relatively coarse grid. The grid arrangement consists of 16 points with a grid distance of 180 nautical miles. Figure 3 shows the arrangement. This grid represents the Gulf fairly well, but has the disadvantage that, in areas of large depth gradient, the terms in the equations may not occur with the relative magnitudes desired for a relaxation solution.

Applying equations (12) and appropriate boundary conditions to each of the 16 grid points produces a system of 32 equations for 32 unknown tidal heights. Of these, eight do not contain the dominant coefficients desired for relaxation. Six of these eight are for interior points with eight terms in each equation. This means that successful reduction of the residuals of these equations, especially in the later stages of the solution, may depend upon systematic adjustment of the number of tidal heights to avoid deleterious effects on the residuals at adjacent points.

In every iteration values of the tidal heights used were the results of the last calculations made at the point. Some systems (e.g., the method of Richardson as described by Hildebrand [ibid.]) are cyclic in nature using only calculations from previous iterations. The approach in this work is not

cyclic, but is based upon maximum reduction of the largest residuals consistent with the long-range effect upon adjacent points.

Initial passes were made based on a first guess field derived from Grace's results and observational data reported by Marmer. Over-relaxation techniques were utilized with λ taking values from 1.5 to 1.2. The chief drawback of this technique is that it can produce oscillations in the values of the tidal heights and thus fail to converge. At the first sign of an undamped oscillation, λ was lowered to 1. On subsequent iterations the relaxation factor was varied to meet the needs of each equation. To determine corrections late in the calculations systems of simultaneous equations were solved for the appropriate values of λ. At this stage the equations were very sensitive, and large corrections frequently had deleterious effects on adjacent points. To most efficiently deal with these situations tabulations of present residuals and the effects of proposed changes in tidal heights upon them were necessary. These were most helpful in determining the interaction of changes in the tidal heights. After twenty iterations component tidal heights (ζ_1 and ζ_2) at all interior and solid boundary points were defined to an accuracy of plus or minus three centimeters. Those at the Yucatan Passage and Straits of Florida were extremely sensitive to values of input current, fluctuating approximately ten centimeters with changes of only

one centimeter per second in the associated velocity component. The effects of these points on other equations is not large. Since the current velocities utilized are not considered sufficiently accurate to warrant it, these points were not relaxed to the degree applied to the other points in the grid.

The results of these calculations are contained in Appendix D. In general, this solution was very similar to that accomplished using the computer on the same 16 point grid. The maximum difference in magnitude of tidal height was 9.06 cms. The average difference was 3.23 cms. This computer solution is discussed in more detail below.

6. Solution by Digital Computer

Adaptation of the finite difference formulae (12) to the computer is a difficult task. For efficient iteration a procedure is necessary for expressing all equations in a common format. This procedure permits the use of indexing operations in the identification and location of equations. This utilization of indexing is accomplished by the inclusion of identification of adjacent points and their relative positions in the input data. Through this, and a coded method for points outside the boundary the program identifies interior and specific boundary locations and simultaneously applies the proper equations to them. Fortran 60 program language is used. Based on the needs of this particular application, convenient computer capacity, and the concept of versatility, the maximum number of points in the grid is 1200. These points are numbered serially, and points outside the boundary are identified by numbers greater than 1200. Since the requirement for ficticious tidal heights outside the boundary has been eliminated (see above), the only data required outside the boundary are tidal current velocities and basin depths.

The presence of an identification number greater than 1200 then specifies a particular boundary and two such numbers specify a particular corner. The current velocities included as data for points outside the boundaries replace tidal heights for those points in the equations at the boundary point. This

requires alteration of the coefficients in the equations to fit the location of the point at which the equations are being applied. This is accomplished from the general forms,

$$
\begin{aligned}
(\zeta_1 - H)_0 = \Big[& a_1 (\zeta_1 - H)_{X+1} && - b_1 (\zeta_2)_{X+1} \\
& + a_2 (\zeta_1 - H)_{X-1} && + b_2 (\zeta_2)_{X-1} \\
& + a_3 (\zeta_1 - H)_{Y+1} && + b_3 (\zeta_2)_{Y+1} \\
& + a_4 (\zeta_1 - H)_{Y+1} && - b_4 (\zeta_2)_{Y-1} \\
& + c_1 \Big] / C2
\end{aligned}
\tag{21}
$$

and

$$
\begin{aligned}
(\zeta_2)_0 = \Big[& a_1 (\zeta_2)_{X+1} && + b_1 (\zeta_1 - H)_{X+1} \\
& + a_2 (\zeta_2)_{X-1} && - b_2 (\zeta_1 - H)_{X-1} \\
& + a_3 (\zeta_2)_{Y+1} && - b_3 (\zeta_1 - H)_{Y+1} \\
& + a_4 (\zeta_2)_{Y-1} && + b_4 (\zeta_1 - H)_{Y-1} \Big] / C2 .
\end{aligned}
$$

Thus the coefficients depend upon the location of the point. Appendix A presents these coefficients in detail and the computer program is given in Appendix C.

The system of equations derived from the grid utilized in the hand calculations was then solved using the computer. Actual depths and currents were employed. This system converged in 37 iterations.

In a study of this nature results are best displayed by the analysis of the solution to obtain cotidal and corange lines. Cotidal lines are lines along which the high tide occurs simultaneously. Corange lines are lines along which the tidal amplitudes are equal. As set forth above these results apply to the M2 constituent, and the phases are relative to the passage of the moon through the 90th west meridian. The lack of detail

in this solution causes the corange and cotidal analysis to be somewhat subjective. The analysis of the 16 point solution is illustrated in Figure 4 which is based upon coastal data as well as the computer solution. It is evident that most of the distortion of the tides occurs in the coastal regions. This does not mean that the solution is in error. The lack of detail does not permit identification of a small-scale amphidromy such as that proposed by Grace or seen in the solution discussed below. The path of the progressive tidal wave indicated by the cotidal lines is logical and amplitude distributions agree with observed coastal data.

To obtain more detail in the solution a finer grid was desired. Using a grid distance of thirty nautical miles a grid was then constructed in an irregular shape closely approximating the Gulf. Real depths were used, taken from U. S. Coast and Geodetic Survey Chart 1007. The grid consisted of 504 points. Current data mentioned above were used in the Straits of Florida and Yucatan Strait. The resulting system of equations failed to converge after 2500 iterations. In the initial iterations the tidal heights increased slowly in magnitude. After 25 iterations the solutions were unrealistic and they increased more rapidly in magnitude. Computational instability seemed to be present and further efforts were directed at a more tractable situation.

Using the same grid a constant depth of 1000 fathoms

was introduced. The system of equations converged in 378 iterations with a relative convergence criterion of 0.1. That is, each tidal height within the grid must change by less than 10% of that of the previous iteration before the iterative procedure is stopped. The maximum change of tidal height over the last three iterations was .12 centimeters. These results are tabulated in Appendix D. Figure 5 shows the analysis of these results in terms of corange and cotidal lines. The path of the M2 constituent tidal wave crest through the Gulf is described by the cotidal lines labeled in hours with zero being the time of the passage of the moon through the 90th west meridian.

The detail obtained in a 504 point grid considerably lessens subjectivity in these cotidal and corange analyses. Significant aspects of these analyses are: the well-developed amphidromy in the Northeastern corner of the Gulf, the presence of the greater amplitudes around the coast, the presence of a rather flat field of low amplitudes in the vicinity of the amphidromic point, and the relatively high amplitudes in areas of known semi-diurnal dominance. The path of the tidal wave crest through the Gulf is very well defined.

Both the 16- and 504- point grid computer solutions produce corange and cotidal lines consistent with coastal data. Because one is based upon constant depth and the other upon actual depths the analyses differ considerably in some areas;

however, the regime shifts mentioned in Section 3 (say, semidiurnal to diurnal) are clearly indicated in both Figures 4 and 5.

7. Conclusions and Acknowledgements

The three solutions obtained indicate that the numerical methods previously discussed are effective in predicting tides at sea. Based on the relatively quick convergence with actual depths of the 16 point system which has a grid distance of 180 nautical miles, the convergence with constant depth of the 504 point system which has a grid distance of 30 miles, and the failure to converge with actual depths of the 504 point system, it is reasonable to assume that there exists an intermediate grid distance such that convergence may be obtained with maximum detail in the solution. Another approach is to obtain the maximum detail within the field by mathematically smoothing the actual depth data to such a degree that the 504 point system would converge. The danger here is that excessive smoothing could distort the data thus giving less realism in the solution.

The cotidal and corange analyses of the constant and variable-depth models indicate that variations in depth are not solely responsible for the tidal regime shifts briefly described in Section 1 (see Figure 4). Additional studies may deliniate the relative importances of spacial forcing function variation, input current, basin shape, and basin depths upon tidal amplitude distributions and perhaps uncover other important variables.

By removing certain topographical features of the Gulf floor from the data it would be possible to assess their effect upon

tidal distributions. Similarly variations in input currents and the forcing function field could be examined for their effects. By adjusting input currents only, under a condition of maximum realism and detail in all other data, to obtain the constituent distribution most compatible with coastal observations one would be able to evaluate the accuracy of present constituent tidal current data.

The only geographical limitation in this technique is the size of the basin due to the beta-plane assumption. Thus all other seas comparable in size to the Gulf of Mexico are well suited for an investigation of this sort.

This study and the writer have both benefitted greatly from the interest, guidance and considerable abilities of members of the faculty and staff of the United States Naval Postgraduate School. Principal among these are: Associate Professor T. Green, III for his encouragement, help, considerable effort and constructive guidance; Professor G. J. Haltiner for his counsel and personal example of scientific excellence; Associate Professor J. B. Wickham for his incisive and constructive criticism; R. D. Brunell for his personal interest, efforts in the programming and technical assistance; also Dr. G. B. Austin and Dr. G. Dowling of the U. S. Navy Mine Defense Laboratory, Panama City, Florida for their interest and assistance in selection of this topic; and Mrs. Julene L. Gainer for her patience, encouragement, understanding and recently acquired proficiency at the desk calculator.

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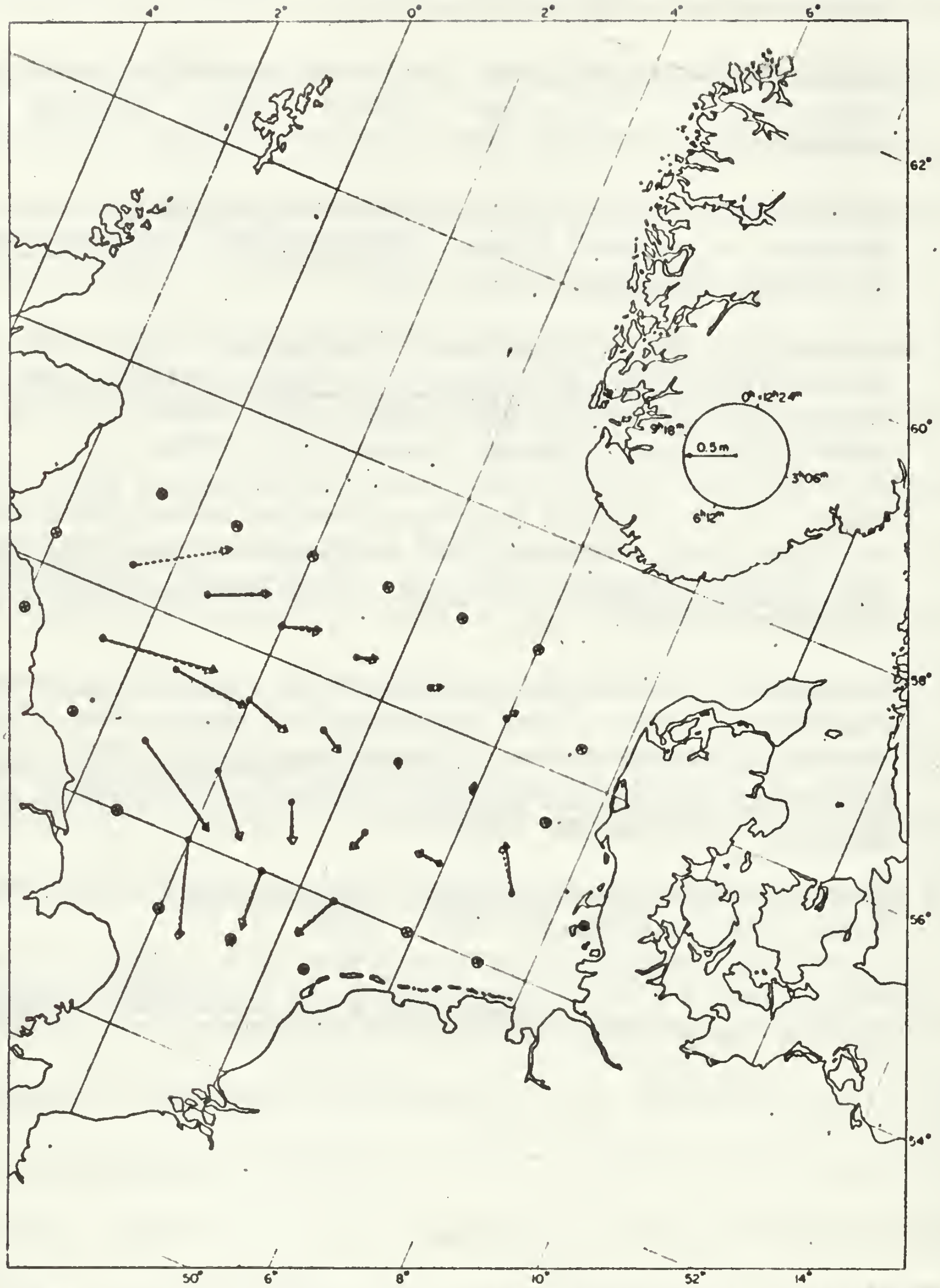


Fig. 1. Representation of amplitude and phase of the M_2 constituent in the southern North Sea. The length of the arrows is proportional to the amplitude, and the direction is determined by the phase. Full arrows after computation; broken arrows after observations. From Hansen (2).

26—s. I.

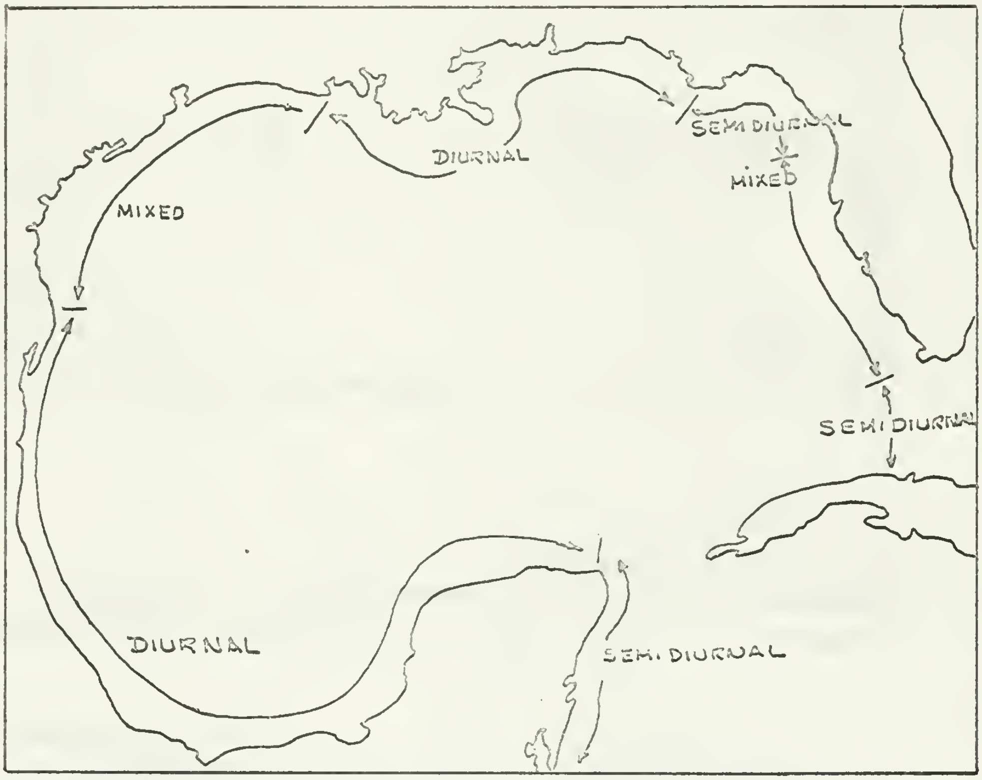


Figure 2 - GULF OF MEXICO TIDAL REGIMES

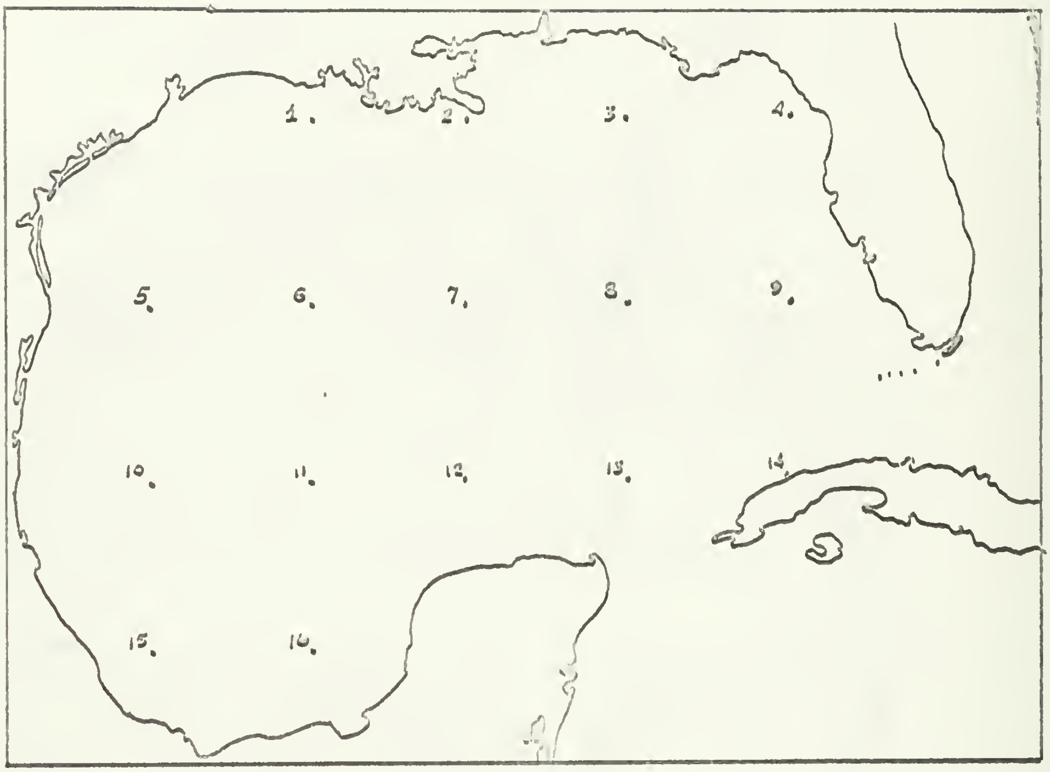


Figure 3. - GRID ARRANGEMENT FOR HAND CALCULATIONS

This consists of 16 point grid with 180 nautical mile grid distance. Input currents were u at point 14 and v at point 13.

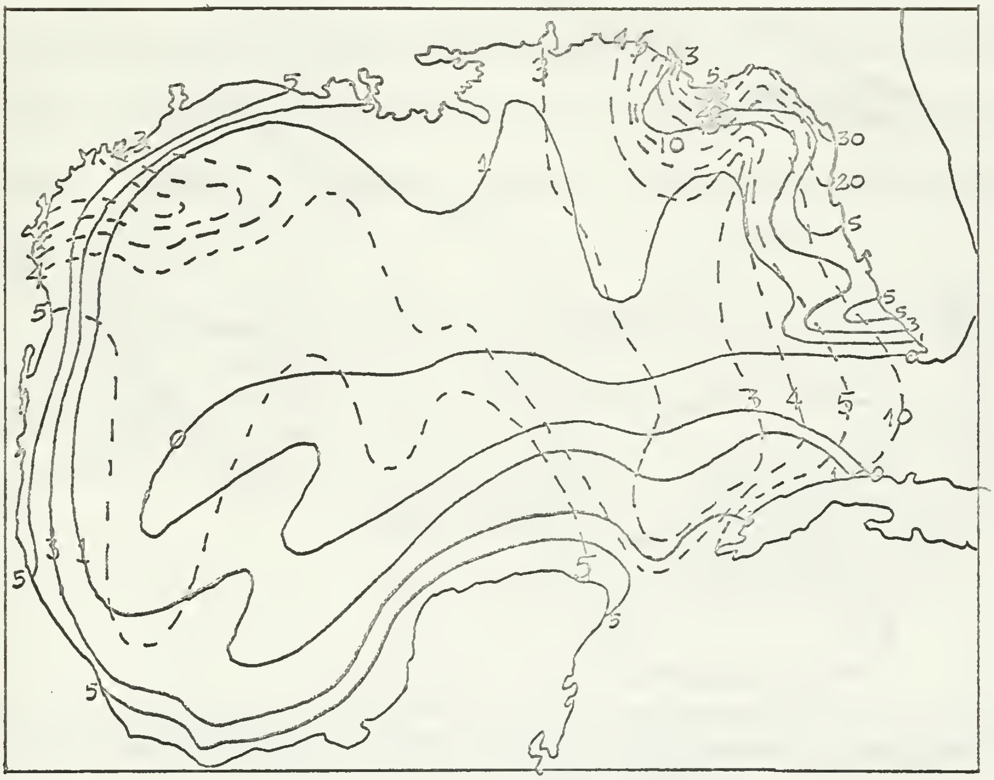


Figure 4. - RESULTS OF VARIABLE DEPTH COMPUTER SOLUTION:

Corange lines (dashed) and cotidal lines (solid). Corange lines are labeled in centimeters. Cotidal lines are labeled in degrees denoting the phase of the tide relative to meridional passage of the moon through 90°W.

Figure 5. - RESULTS OF CONSTANT DEPTH COMPUTER SOLUTION:

Corange lines (dotted) labeled in centimeters, Cotidal lines (solid) labeled in hours with zero being the time of the moon's passage through the 90th West meridian.

APPENDIX A

FINITE DIFFERENCE EQUATIONS FOR INTERIOR AND ALL BOUNDARY POINTS ADAPTED FOR COMPUTER SOLUTION

Equation (12) represents the finite difference form for an interior point. We have shown that at each boundary unique modifications of this equation are necessary to represent continuity and the boundary conditions. To avoid the necessity of separately establishing each equation within the program and to create a compact iteration scheme the following system was devised.

Examination of all the equations led to a general form,

$$\begin{aligned}(\mathcal{S}_1 - H)_0 = {} & a_1(\mathcal{S}_1 - H)_{X+1} - b_1(\mathcal{S}_2)_{X+1} \\ & + a_2(\mathcal{S}_1 - H)_{X-1} + b_2(\mathcal{S}_2)_{X-1} \\ & + a_3(\mathcal{S}_1 - H)_{Y+1} + b_3(\mathcal{S}_2)_{Y+1} \\ & + a_4(\mathcal{S}_1 - H)_{Y-1} - b_4(\mathcal{S}_2)_{Y-1} \\ & + C_1 \,/\, C2\end{aligned}$$

where the symbol / indicates that all terms on the right side are divided by C2

and

$$\begin{aligned}(\mathcal{S}_2)_0 = {} & a_1(\mathcal{S}_2)_{X+1} + b_1(\mathcal{S}_1 - H)_{X+1} \\ & + a_2(\mathcal{S}_2)_{X-1} - b_2(\mathcal{S}_1 - H)_{X-1} \\ & + a_3(\mathcal{S}_2)_{Y+1} - b_3(\mathcal{S}_1 - H)_{Y+1} \\ & + a_4(\mathcal{S}_2)_{Y-1} + b_4(\mathcal{S}_1 - H)_{Y-1} \\ & /C2 .\end{aligned}$$

These equations are set up to carry out Gauss-Seidel iteration, but modification for other solution methods could be made at this point.

Now by indexing within the program the constants may be called to fit any of the previously derived equations satisfying both dynamic considerations and boundary conditions. For interior points the constants are:

$$a_1 = 4h_0 + h_{x+1} - h_{x-1} \qquad b_1 = \frac{F}{g}\left[h_{y+1} - h_{y-1}\right]$$

$$a_2 = 4h_0 - h_{x+1} + h_{x-1} \qquad b_2 = \frac{F}{g}\left[h_{y+1} - h_{y-1}\right]$$

$$a_3 = 4h_0 + h_{y+1} - h_{y-1} \qquad b_3 = \frac{F}{g}\left[h_{x+1} - h_{x-1}\right]$$

$$a_4 = 4h_0 - h_{y+1} + h_{y-1} \qquad b_4 = \frac{F}{g}\left[h_{x+1} - h_{x-1}\right]$$

$$c_1 = H\frac{4\epsilon^2(\sigma^2 - F^2)}{g} \qquad c_2 = 16h_0 - \frac{\epsilon^2(\sigma^2 - F^2)}{g}$$

For the West Boundary,

$$a_1 = 8h_0 \qquad b_1 = 0$$

$$a_2 = \frac{2\epsilon(\sigma^2 - F^2)F}{g\sigma^2}(h_{y+1} - h_{y-1}) \qquad b_2 = -\frac{2\epsilon(\sigma^2 - F^2)(4h_0 - h_{x+1} + h_{x-1})}{g\sigma}$$

$$a_3 = 4h_0 + (h_{y+1} - h_{y-1})(1 - F^2/\sigma^2) \qquad b_3 = 4\frac{Fh_0}{\sigma}$$

$$a_4 = 4h_0 - (h_{y+1} + h_{y-1})(1 - F^2/\sigma^2) \qquad b_4 = 4\frac{Fh_0}{\sigma}$$

$$c_1 = H\frac{4\epsilon^2(\sigma^2 - F^2)}{g} \qquad c_2 = 16h_0 - \frac{4\epsilon^2(\sigma^2 - F^2)}{g}$$

East Boundary,

$$a_1 = \frac{2\epsilon(\sigma^2 - F^2)F}{g\sigma^2}(h_{y+1} - h_{y-1}) \qquad b_1 = \frac{2\epsilon(\sigma^2 - F^2)}{g\sigma}(4h_0 + h_{x+1} - h_{x-1})$$

$$a_2 = 8h_0 \qquad b_2 = 0$$

$$a_3 = 4h_0 + (h_{y+1} - h_{y-1})(1 - F^2/\sigma^2) \qquad b_3 = -\frac{4Fh_0}{\sigma}$$

$$a_4 = 4h_0 - (h_{y+1} - h_{y-1})(1 - F^2/\sigma^2) \qquad b_4 = -\frac{4Fh_0}{\sigma}$$

$$c_1 = \frac{4\epsilon^2(\sigma^2 - F^2)H}{g} \qquad c_2 = 16h_0 - 4\epsilon^2\frac{(\sigma^2 - F^2)}{g}$$

South Boundary,

$$a_1 = 4h_o + (h_{x+1} - h_{x-1})(1 - F^2/\sigma^2) \qquad b_1 = 4h_oF/\sigma$$

$$a_2 = 4h_o - (h_{x+1} - h_{x-1})(1 - F^2/\sigma^2) \qquad b_2 = 4h_oF/\sigma$$

$$a_3 = 8h_o \qquad b_3 = 0$$

$$a_4 = \frac{2\epsilon(\sigma^2 - F^2)F}{g\sigma^2}(h_{x+1} - h_{x-1}) \qquad b_4 = \frac{2\epsilon(\sigma^2 - F^2)}{g\sigma}(4h_o - h_{y+1} + h_{y-1})$$

$$c_1 = H\,\frac{4\epsilon^2(\sigma^2 - F^2)}{g} \qquad c_2 = 16h_o - 4\epsilon^2(\sigma^2 - F^2)/g$$

North Boundary,

$$a_1 = 4h_o + (h_{x+1} - h_{x-1})(1 - F^2/\sigma^2) \qquad b_1 = \frac{-4h_oF}{\sigma}$$

$$a_2 = 4h_o - (h_{x+1} - h_{x-1})(1 - F^2/\sigma^2) \qquad b_2 = \frac{-4h_oF}{\sigma}$$

$$a_3 = \frac{-2\epsilon(\sigma^2 - F^2)F(h_{x+1} - h_{x-1})}{g\sigma^2} \qquad b_3 = \frac{2\epsilon(\sigma^2 - F^2)(4h_o + h_{y+1} - h_{y-1})}{g}$$

$$a_4 = 8h_o \qquad b_4 = 0$$

$$c_1 = \frac{4\epsilon^2(\sigma^2 - F^2)}{g}H \qquad c_2 = 16h_o - \frac{4\epsilon^2(\sigma^2 - F^2)}{g}$$

Southwest corner,

$$a_1 = 8h_o \qquad b_1 = 0$$

$$a_2 = \frac{2\epsilon(\sigma^2 - F^2)F(4h_o)}{g\sigma^2(1 - F^2/\sigma^2)} \qquad b_2 = \frac{2\epsilon(\sigma^2 - F^2)(4h_o)}{g\sigma(1 - F^2/\sigma^2)} + (h_{x+1} - h_{x-1})$$

$$a_3 = 8h_o \qquad b_3 = 0$$

$$a_4 = -\frac{8\epsilon(\sigma^2 - F^2)Fh_o}{g\sigma^2(1 - F^2/\sigma^2)} \qquad b_4 = \frac{2\epsilon(\sigma^2 - F^2)}{g\sigma}\left(4h_o - h_{y+1} + h_{y-1} + \frac{F^2}{\sigma^2}\frac{4h_o}{(1 - F^2/\sigma^2)}\right)$$

$$c_1 = \frac{4\epsilon^2 H(\sigma^2 - F^2)}{g} \qquad c_2 = 16h_o - \frac{4\epsilon^2(\sigma^2 - F^2)}{g}$$

Northwest corner,

$$a_1 = -\frac{F\,4\epsilon h_o(\sigma^2 - F^2)}{g\sigma^2(1 - F^2/\sigma^2)} \qquad b_1 = -\frac{2\epsilon(\sigma^2 - F^2)}{g\sigma}\left(\frac{4h_o}{1 - F^2/\sigma^2} + h_{x+1} - h_{x-1}\right)$$

$$a_2 = 8h_o \qquad b_2 = 0$$

$$a_3 = \frac{-F\,4\epsilon h_o(\sigma^2 - F^2)}{g\sigma^2(1 - F^2/\sigma^2)} \qquad b_3 = \frac{2\epsilon(\sigma^2 - F^2)\left(4h_o + h_{y+1} - h_{y-1} + \frac{F^2}{\sigma^2}\frac{4h_o}{(1 - F^2/\sigma^2)}\right)}{g\sigma}$$

$$a_4 = 8h_o \qquad b_4 = 0$$

$$c_1 = \frac{4\epsilon^2 H(\sigma^2 - F^2)}{g} \qquad c_2 = 16h_o - \frac{4\epsilon^2(\sigma^2 - F^2)}{g}$$

Southeast corner,

$$a_1 = \frac{8\epsilon F h_o(\sigma^2 - F^2)}{g\sigma^2(1 - F^2/\sigma^2)} \qquad b_1 = \frac{2\epsilon(\sigma^2 - F^2)(4h_o + h_{x+1} - h_{x-1})}{g\sigma}$$

$$a_2 = 8h_o \qquad b_2 = 0$$

$$a_3 = 8h_o \qquad b_3 = 0$$

$$a_4 = \frac{8\epsilon h_o F(\sigma^2 - F^2)}{g\sigma^2(1 - F^2/\sigma^2)} \qquad b_4 = \frac{2\epsilon(\sigma^2 - F^2)\left(4h_o - h_{y+1} + h_{y-1} + \frac{F^2}{\sigma^2}\frac{4h_o}{(1 - F^2/\sigma^2)}\right)}{g\sigma}$$

$$C_1 = \frac{4\epsilon^2(\sigma^2 - F^2)\,H}{g} \qquad C_2 = 16h_o - \frac{4\epsilon^2(\sigma^2 - F^2)}{g}$$

Northeast corner,

$$a_1 = \frac{-F\,8h_o\epsilon(\sigma^2 - F^2)}{g\sigma^2(1 - F^2/\sigma^2)} \qquad b_1 = \frac{-2\epsilon(\sigma^2 - F^2)(4h_o + h_{x+1} - h_{x-1})}{g\sigma}$$

$$a_2 = 8h_o \qquad b_2 = 0$$

$$a_3 = -\frac{F}{\sigma}\,\frac{2\epsilon(\sigma^2 - F^2)(4h_o)}{g\sigma(1 - F^2/\sigma^2)} \qquad b_3 = \frac{2\epsilon(\sigma^2 - F^2)\left(4h_o + h_{y+1} - h_{y-1} + \frac{F^2}{\sigma^2}\left(\frac{4h_o}{(1 - F^2/\sigma^2)}\right)\right)}{g\sigma}$$

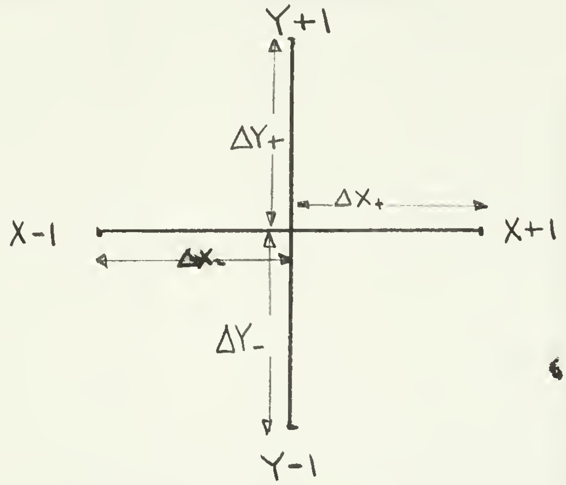
$$a_4 = 8h_o \qquad b_4 = 0$$

$$C_1 = \frac{4\epsilon^2(\sigma^2 - F^2)\,H}{g} \qquad C_2 = 16h_o - \frac{4\epsilon^2(\sigma^2 - F^2)}{g}.$$

APPENDIX B

DIFFERENCING EQUATIONS FOR VARIABLE GRID DISTANCE

Given a relative grid arrangement of the sort,



it is desirable to use only contiguous points in the differencing scheme because the use of other points would require a field of values and points outside the real grid. This in turn would negate the use of central differences in evaluating the second derivative.

The basic forward differencing formulae were applied to give

$$\nabla^2 Z = \frac{(\Delta X_-)(Z_{X+1}) + (\Delta X_+)(Z_{X-1}) - Z_0\left[(\Delta X_+) + (\Delta X_-)\right]}{(\Delta X_-)^2(\Delta X_+)} + \frac{(\Delta Y_-)(Z_{Y+1}) + (\Delta Y_+)(Z_{Y-1}) - Z_0\left[(\Delta Y_+) + (\Delta Y_-)\right]}{(\Delta Y_-)^2\, \Delta Y_+}$$

where Z is a dummy variable. This expression is quite involved and the stability of calculation about such a point is highly questionable.

APPENDIX C

THE COMPUTER PROGRAM

```
      PROGRAM TIDES
      DIMENSION N(1200,4),CON(1200,10),FF(1200),ZL(1200),HF(1200),
     1WL(1200),DEPTH(1401),Z(1401),W(1401)
      COMMON N,CON,FF,ZL,HF,WL,DEPTH,Z,W
      EQUIVALENCE (FF,ZL),(DEPTH,WL)
      READ 10,NN,NB,IMAX,IPRINT
   10 FORMAT(7I10)
      READ 15,SIG,G,EP,TSIO,EP1,OMEGA,XK
   15 FORMAT(7E10.3)
      READ20,(I,N(I,1),N(I,2),N(I,3),N(I,4),DEPTH(I),FF(I),HF(I),J=1,NN)
   20 FORMAT(5I5,3E10.3)
      DO 25 I=1,NN
      FF(I)=0.0174533*FF(I)
      HF(I)=0.0174533*HF(I)
   25 CONTINUE
      READ 30,(I,Z(I),W(I),DEPTH(I),J=1,NB)
   30 FORMAT(I5,3E10.3)
      READ 35, (Z(I),W(I),I=1,NN)
   35 FORMAT(6E10.3)
      NP=NN+NB
      DO 37 I=1,NP
      DEPTH(I)=-182.88*DEPTH(I)
   37 CONTINUE
      DO 40 I=1,NN
      HF(I)=XK*COSF(FF(I))**2*COSF(2.0*(HF(I)-TSIO))
      FF(I)=2.0*OMEGA*SINF(FF(I))
   40 CONTINUE
      DO 42 I=1,NN
      Z(I)=Z(I)-HF(I)
   42 CONTINUE
      DO 1000 I=1,NN
      IF(N(I,1)-1200) 50,50,45
   45 K=2
      GO TO 90
   50 IF(N(I,2)-1200) 60,60,55
   55 K=3
```

```
      GO TO 110
   60 IF(N(I,3)-1200) 70,70,65
   65 K=4
      GO TO 130
   70 IF(N(I,4)-1200)80,80,75
   75 K=5
      GO TO 130
   80 K=1
      GO TO 130
   90 IF(N(I,3)-1200) 100,100,95
   95 K=6
      GO TO 130
  100 IF(N(I,4)-1200) 130,130,105
  105 K=7
      GO TO 130
  110 IF(N(I,3)-1200) 120,120,115
  115 K=8
      GO TO 130
  120 IF(N(I,4)-1200)130,130,125
  125  K=9
  130 IXM=N(I,1)
      IXP=N(I,2)
      IYM=N(I,3)
      IYP=N(I,4)
      GO TO(140,150,160,170,180,190,200,210,220),K
  140 CXC=DEPTH(IXP)-DEPTH(IXM)
      CYC=DEPTH(IYP)-DEPTH(IYM)
      A1=4.0*EP**2*(SIG**2-FF(I)**2)/G
      CON(I,1)=4.0*DEPTH(I)+CXC
      CON(I,2)=4.0*DEPTH(I)-CXC
      CON(I,3)=DEPTH(I)*4.0+CYC
      CON(I,4)=DEPTH(I)*4.0-CYC
      A2=FF(I)/SIG
      CON(I,5)=A2*CYC
      CON(I,6)=CON(I,5)
      CON(I,7)=A2*CXC
```

```
      CON(I,8)=CON(I,7)
      CON(I,9)=HF(I)*A1
      CON(I,10)=16.0*DEPTH(I)-A1
      GO TO 1000
  150 CYC=DEPTH(IYP)-DEPTH(IYM)
      A1=FF(I)/SIG
      A2=1.0-A1*A1
      XK=2.0*EP*(SIG**2-FF(I)**2)/G
      CON(I,1)=8.0*DEPTH(I)
      CON(I,2)=XK*FF(I)/SIG**2*CYC
      CON(I,3)=4.0*DEPTH(I)+CYC*A2
      CON(I,4)=DEPTH(I)*4.0-CYC*A2
      CON(I,5)=0.0
      CON(I,6)=-XK/SIG*(4.0*DEPTH(I)-DEPTH(IXP)+DEPTH(IXM))
      CON(I,7)=A1*DEPTH(I)*4.0
      CON(I,8)=CON(I,7)
      A3=2.0*EP*XK
      CON(I,9)=HF(I)*A3
      CON(I,10)=16.0*DEPTH(I)-A3
      GO TO 1000
  160 CYC=DEPTH(IYP)-DEPTH(IYM)
      A1=FF(I)/SIG
      A2=1.0-A1*A1
      XK=2.0*EP*(SIG**2-FF(I)**2)/G
      CON(I,1)=XK*A1/SIG*CYC
      CON(I,2)=2.0*DEPTH(I)*4.0
      A3=A2*CYC
      CON(I,3)=DEPTH(I)*4.0+A3
      CON(I,4)=4.00*DEPTH(I)-A3
      CON(I,5)=-XK/SIG*(4.0*DEPTH(I)+DEPTH(IXP)-DEPTH(IXM))
      CON(I,6)=0.0
      CON(I,7)=-A1*DEPTH(I)*4.0
      CON(I,8)=CON(I,7)
      CON(I,9)=HF(I)*2.0*EP*XK
      CON(I,10)=16.0*DEPTH(I)-2.0*EP*XK
      GO TO 1000
```

```
170 A1=FF(I)/SIG
    A2=1.0-A1*A1
    CXC=DEPTH(IXP)-DEPTH(IXM)
    A3=A2*CXC
    CON(I,1)=DEPTH(I)*4.0+A3
    CON(I,2)=4.0*DEPTH(I)-A3
    CON(I,3)=2.0*DEPTH(I)*4.0
    XK=2.0*EP*(SIG**2-FF(I)**2)/G
    CON(I,4)=-XK*A1/SIG*CXC
    CON(I,5)=A1*DEPTH(I)*4.0
    CON(I,6)=CON(I,5)
    CON(I,7)=0.0
    CON(I,8)=XK/SIG*(DEPTH(I)*4.0-DEPTH(IYP)+DEPTH(IYM))
    CON(I,9)=HF(I)*2.0*EP*XK
    CON(I,10)=16.0*DEPTH(I)-2.0*EP*XK
    GO TO 1000
180 A1=FF(I)/SIG
    A2=1.0-A1*A1
    CXC=DEPTH(IXP)-DEPTH(IXM)
    XK=2.0*EP*(SIG**2-FF(I)**2)/G
    A3=A2*CXC
    CON(I,1)=4.0*DEPTH(I)+A3
    CON(I,2)=DEPTH(I)*4.0-A3
    CON(I,3)=-XK*A1/SIG*CXC
    CON(I,4)=2.0*DEPTH(I)*4.0
    CON(I,5)=-A1*DEPTH(I)*4.0
    CON(I,6)=CON(I,5)
    CON(I,7)=XK/SIG*(DEPTH(I)*4.0+DEPTH(IYP)-DEPTH(IYM))
    CON(I,8)=0.0
    CON(I,9)=HF(I)*2.0*EP*XK
    CON(I,10)=16.0*DEPTH(I)-2.0*EP*XK
    GO TO 1000
190 A1=FF(I)/SIG
    A2=1.0-A1*A1
    XK=2.0*EP* (SIG**2-FF(I)**2)/G
    A3=XK/SIG
```

```
      CON(I,1)=2.0*DEPTH(I)*4.0
      CON(I,2)=A1*A3*DEPTH(I)/A2*4.0
      CON(I,3)=CON(I,1)
      CON(I,4)=-CON(I,2)
      CON(I,5)=0.0
      CON(I,6)=A3*(DEPTH(I)*4.0/A2+DEPTH(IXP)-DEPTH(IXM))
      CON(I,7)=0.0
      CON(I,8)=A3*(DEPTH(I)*4.0-DEPTH(IYP)+DEPTH(IYM)+A1**2*4.0*DEPTH(I)/A2))
     1/A2)
      CON(I,9)=HF(I)*2.0*EP*XK
      CON(I,10)=16.0*DEPTH(I)-2.0*EP*XK
      GO TO 1000
  200 A1=FF(I)/SIG
      A2=DEPTH(I)/(1.0-A1*A1)*4.0
      XK=2.0*EP*(SIG**2-FF(I)**2)/G
      A3=XK/SIG
      CON(I,1)=2.0*DEPTH(I)*4.0
      CON(I,2)=-A1*A3*A2
      CON(I,3)=CON(I,2)
      CON(I,4)=CON(I,1)
      CON(I,5)=0.0
      CON(I,6)=-A3*(A2+DEPTH(IXP)-DEPTH(IXM))
      CON(I,7)=A3*(DEPTH(I)*4.0+DEPTH(IYP)-DEPTH(IYM)+A1*A1*A2)
      CON(I,8)=0.0
      CON(I,9)=HF(I)*2.0*EP*XK
      CON(I,10)=16.0*DEPTH(I)-2.0*EP*XK
      GO TO 1000
  210 A1=FF(I)/SIG
      A2=DEPTH(I)/(1.0-A1*A1)*4.0
      XK=2.0*EP*(SIG**2-FF(I)**2)/G
      A3=XK/SIG
      CON(I,1)=A1*A3*A2
      CON(I,2)=2.0*DEPTH(I)*4.0
      CON(I,3)=CON(I,2)
      CON(I,4)=CON(I,1)
      CON(I,5)= A3*(A2+DEPTH(IXP)-DEPTH(IXM))
```

```
      CON(I,6)=0.0
      CON(I,7)=0.0
      CON(I,8)=A3*(DEPTH(I)*4.0-DEPTH(IYP)+DEPTH(IYM)+A1*A1*A2)
      CON(I,9)=HF(I)*2.0*EP*XK
      CON(I,10)=16.0*DEPTH(I)-2.0*EP*XK
      GO TO 1000
  220 A1=FF(I)/SIG
      A2=DEPTH(I)*4.0/(1.0-A1*A1)
      XK=2.0*EP*(SIG**2-FF(I)**2)/G
      A3=XK/SIG
      CON(I,1)=-A1*A3*A2
      CON(I,2)=2.0*DEPTH(I)*4.0
      CON(I,3)=-CON(I,1)
      CON(I,4)=CON(I,2)
      CON(I,5)=-A3*(A2+DEPTH(IXP)-DEPTH(IXM))
      CON(I,6)=0.0
      CON(I,7)=A3*(DEPTH(I)*4.0+DEPTH(IYP)-DEPTH(IYM)+A1*A1*A2)
      CON(I,8)=0.0
      CON(I,9)=2.0*EP*XK*HF(I)
      CON(I,10)=16.0*DEPTH(I)-2.0*EP*XK
 1000 CONTINUE
      DO 915 I=1,NN
  915 CONTINUE
      IJK=0
 1020 DO 1010 I=1,NN
      ZL(I)=Z(I)
      WL(I)=W(I)
 1010 CONTINUE
      IJK=IJK+1
      DO 1030 I=1,NN
      IXM=N(I,1)
      IXP=N(I,2)
      IYM=N(I,3)
      IYP=N(I,4)
      Z(I)=(Z(IXP)*CON(I,1)+Z(IXM)*CON(I,2)+Z(IYP)*CON(I,3)+Z(IYM)*CON(I
     1,4)-W(IXP)*CON(I,5)+W(IXM)*CON(I,6)+W(IYP)*CON(I,7)-W(IYM)*CON(I,8
```

```
     2)+CON(I,9))/CON(I,10)
      W(I)=(W(IXP)*CON(I,1)+W(IXM)*CON(I,2)+W(IYP)*CON(I,3)+W(IYM)*CON(I
     1,4)+Z(IXP)*CON(I,5)-Z(IXM)*CON(I,6)-Z(IYP)*CON(I,7)+Z(IYM)*CON(I,8
     2))/CON(I,10)
 1030 CONTINUE
      ITL=1
      DO 1060 I=1,NN
      IF(ABSF((ZL(I)-Z(I))/Z(I))-EP1) 1050,1050,1040
 1040 ITL=2
      GOTO 1070
 1050 IF(ABSF((WL(I)-W(I))/W(I))-EP1) 1060,1060,1055
 1055 ITL=2
      GO TO 1070
 1060 CONTINUE
      GO TO 1100
 1070 IF(IJK-IMAX) 1080,1075,1075
 1075 ITL=3
      GO TO 1100
 1080 IF(XMODF(IJK,IPRINT)) 1020,1100,1020
 1100 GO TO(1130,1180,1110),ITL
 1110 PRINT 1120
 1120 FORMAT(1H1,5X,28HMAXIMUM ITERATIONS EXCEEDED )
 1130 DO 1140 I=1,NN
      Z(I)=Z(I)+HF(I)
 1140 CONTINUE
      PRINT 1150,IJK
 1150 FORMAT(1H1,38X,39HFINAL RESULTS     NUMBER OF ITERATIONS ,I5/15X,
     15H NODE,10X,5HTSI 1,15X,5HTSI 2,12X,9HMAGNITUDE,9X,9HDIRECTION  )
      DO 1160 I=1,NN
      TEMP=SQRTF(Z(I)**2+W(I)**2)
      TEP=57.29582*ACTAN(W(I),Z(I))
      PRINT 1170,I,Z(I),W(I),TEMP,TEP
      WRITE OUTPUT TAPE 2,1175,I,Z(I),W(I),TEMP,TEP
 1175 FORMAT(I5,4E18.7)
 1160 CONTINUE
 1170 FORMAT(15X,I5,5X,4E20.8)
```

```
      STOP
 1180 PRINT 1190,IJK
 1190 FORMAT(1H1,5X,38HINTERMEDIATE RESULTS, ITERATION NUMBER ,I5 ,/10X,
     15HNODE,10X,5HTSI 1,20X,5HTSI 2 )
      PRINT 1200,(I,Z(I),W(I),I=1,NN)
 1200 FORMAT(10X,I5,2E20.8)
      GO TO 1020
      END
      FUNCTION ACTAN(A,B)
      PI=3.141592654
      PID2=1.570796326
      IF(A) 90,10,50
   10 IF(B) 40,20,30
   20 PRINT 25,A,B
   25 FORMAT(//4X,39HIMPOSSIBLE CALCULATION IN ACTAN.    A= E18.8,5X,
     13HB= ,E18.8)
   30 ACTAN=0.0
      GO TO 200
   40 ACTAN=PI
      GO TO 200
   50 IF(B) 80,60,70
   60 ACTAN=PID2
      GO TO 200
   70 ACTAN=ATANF(A/B)
      GO TO 200
   80 ACTAN=PI+ATANF(A/B)
      GO TO 200
   90 IF(B) 110,100,70
  100 ACTAN=-PID2
      GO TO 200
  110 ACTAN=-PI+ATANF(A/B)
  200 RETURN
      END
      END
```

APPENDIX D

THE NUMERICAL RESULTS OF THE 504 POINT CONSTANT DEPTH SOLUTION

POINT	MAGNITUDE (cms)	PHASE (Degrees)
1.	1.3887000	-141.24518
2.	1.3756521	-138.80641
3.	1.3217907	-131.90854
4.	1.2539393	-120.89377
5.	1.2029531	-105.43202
6.	1.2059750	-86.843942
7.	1.2334819	-73.334869
8.	1.7517582	-172.50711
9.	1.7194041	-172.30618
10.	1.7070080	-171.97713
11.	1.6958950	-171.56044
12.	1.3027698	-140.26361
13.	1.2262120	-137.87719
14.	1.1332772	-130.61788
15.	1.0404123	-118.38878
16.	.98849004	-99.810982
17.	1.0718649	-.75.709188
18.	1.4355376	-53.916750
19.	2.4490565	-39.475631
20.	4.1602358	-23.200350
21.	7.4439120	-4.1130445

22.	5.4827457	-2.1818875
23.	1.6776942	-172.36824
24.	1.6361282	-172.15673
25.	1.5939419	-171.76625
26.	1.5654488	-171.39348
27.	1.5564011	-170.96102
28.	1.5592573	-170.38945
29.	1.5838993	-169.61342
30.	1.5963884	-167.86595
31.	1.5907292	-165.26372
32.	1.5549091	-161.59242
33.	1.4619722	-156.27691
34.	1.2553052	-148.15227
35.	1.1279148	-142.15830
36.	1.0039504	-134.12128
37.	.88153194	-131.44064
38.	.79735342	-100.55519
39.	.86428115	-71.147404
40.	1.2173230	-45.294358
41.	1.9446390	-28.377334
42.	2.9908028	-15.251722
43.	4.1469681	-3.9740857
44.	3.7697796	1.7862483
45.	2.9949950	9.8396774
46.	1.5900268	-171.85187

47.	1.5265542	-171.55308
48.	1.4871450	-171.29280
49.	1.4817022	-171.10465
50.	1.4258745	-170.47495
51.	1.4202776	-169.98678
52.	1.4242894	-169.33126
53.	1.4326960	-168.36735
54.	1.4367689	-166.84673
55.	1.4264928	-164.70118
56.	1.3910475	-161.82801
57.	1.3153951	-158.02929
58.	1.1943604	-153.24541
59.	1.0752430	-148.40719
60.	.94189596	-142.03557
61.	.79343153	-132.02672
62.	.64595142	-113.55350
63.	.59415964	-79.717909
64.	.80011771	-44.336330
65.	1.2755715	-22.528501
66.	1.8856044	-9.0438157
67.	2.4422500	1.3021725
68.	2.5285018	10.436640
69.	2.5386045	20.977383
70.	1.5837556	-171.35588
71.	1.5189584	-171.12447

72.	1.4459243	-170.90635
73.	1.3873289	-170.62673
74.	1.3523482	-170.37487
75.	1.3203357	-170.00174
76.	1.3014904	-169.57599
77.	1.3007594	-169.12103
78.	1.3081634	-168.52707
79.	1.3162539	-167.68322
80.	1.3231527	-166.52922
81.	1.3207365	-165.02452
82.	1.3008327	-163.15918
83.	1.2537422	-160.91726
84.	1.1791414	-158.41437
85.	1.0871412	-155.84092
86.	.9684444	-152.64950
87.	.82084260	-148.05767
88.	.63568331	-139.73622
89.	.42310620	-119.25154
90.	.32467912	-66.502209
91.	.55898858	-18.151299
92.	.93149353	2.2351259
93.	1.2945355	15.742440
94.	1.5414282	29.348534
95.	1.8510571	43.251027
96.	1.4426962	-170.27041

97.	1.3801700	-170.11656
98.	1.3135581	-169.87776
99.	1.2602636	-169.60957
100.	1.2306716	-169.39582
101.	1.2046669	-169.06696
102.	1.1925103	-168.71396
103.	1.1990822	-168.38716
104.	1.2142370	-167.98880
105.	1.2307824	-167.44851
106.	1.2495502	-166.79710
107.	1.2644726	-166.04208
108.	1.2694225	-165.22816
109.	1.2565822	-164.40579
110.	1.2247704	-163.71041
111.	1.1738689	-163.24897
112.	1.0947432	-162.92580
113.	.98741067	-162.80156
114.	.84121204	-162.89192
115.	.64327279	-163.50925
116.	.40957519	-167.70662
117.	.16316727	160.04043
118.	.27612851	75.916858
119.	.60679389	67.911211
120.	.98720807	74.397350
121.	1.4865882	81.249956

122.	1.3192293	-168.98529
123.	1.2570372	-168.94400
124.	1.1937553	-168.73472
125.	1.1450593	-168.49526
126.	1.1211817	-168.34803
127.	1.1020564	-168.10229
128.	1.0981462	-167.88417
129.	1.1145761	-167.77086
130.	1.1415354	-167.67201
131.	1.1725985	-167.54610
132.	1.2102442	-167.46968
133.	1.2500523	-167.47233
134.	1.2873929	-167.60798
135.	1.3153326	-167.92557
136.	1.3320425	-168.52434
137.	1.3344956	-169.47107
138.	1.3135021	-170.77450
139.	1.2716266	-172.57444
140.	1.2020871	-175.05351
141.	1.0981960	-178.68367
142.	.9868950	175.47563
143.	.88220144	165.61313
144.	.86646759	150.92426
145.	.98653169	136.05371
146.	1.2799964	126.90135

147.	1.7474118	121.69064
148.	1.2092843	-167.64318
149.	1.1461983	-167.67550
150.	1.0849542	-167.48544
151.	1.0403817	-167.27501
152.	1.0224132	-167.21450
153.	1.0109213	-167.08640
154.	1.0166413	-167.05710
155.	1.0451478	-167.22489
156.	1.0872336	-167.49308
157.	1.1373998	-167.83048
158.	1.1995002	-168.32647
159.	1.2705283	-169.00223
160.	1.3470828	-169.88820
161.	1.4228629	-171.00051
162.	1.49577548	-172.38357
163.	1.5614789	-174.05059
164.	1.6100079	-175.98955
165.	1.6439044	-178.23670
166.	1.6564831	179.18471
167.	1.6410146	176.18332
168.	1.6223832	172.66604
169.	1.6032476	168.28565
170.	1.6392330	161.61001
171.	1.7547253	155.14506

172.	2.0045504	150.74902
173.	2.3098794	149.75370
174.	3.0581112	155.39824
175.	1.1086753	-166.33404
176.	1.0442983	-166.35663
177.	.98475467	-166.13134
178.	.94421575	-165.92555
179.	.93236154	-165.95960
180.	.92907261	-165.97488
181.	.94545533	-166.17686
182.	.98770361	-166.67282
183.	1.0474125	-167.33990
184.	1.1201448	-168.13757
185.	1.2111263	-169.14457
186.	1.3187259	-170.35560
187.	1.4407064	-171.76804
188.	1.5713587	-173.36616
189.	1.7083588	-175.15459
190.	1.8462118	-177.11356
191.	1.9728861	-179.21124
192.	2.0883663	178.57807
193.	2.1825526	176.30378
194.	2.2449039	174.02875
195.	2.2967387	172.02866
196.	2.3377767	171.09147

197.	2.4036320	165.81038
198.	2.5284900	162.15293
199.	2.7636824	159.89052
200.	3.1167505	158.85404
201.	3.5327726	157.76749
202.	1.0129899	-165.13166
203.	.94788364	-164.99387
204.	.89049944	-164.63719
205.	.85424915	-164.38986
206.	.84869031	-164.51702
207.	.85389577	-164.69366
208.	.88147957	-165.15934
209.	.93842593	-166.01340
210.	1.0173279	-167.08388
211.	1.1149818	-168.30477
212.	1.2381205	-169.73308
213.	1.3866221	-171.33510
214.	1.5595657	-173.08435
215.	1.7520227	-174.95054
216.	1.9616174	-176.92430
217.	2.1814858	-178.97981
218.	2.3965146	178.91438
219.	2.6017553	176.79859
220.	2.7802197	174.72718
221.	2.9135588	172.74419

222.	3.0161880	170.95484
223.	3.0826239	169.29360
224.	3.1604651	167.06122
225.	3.2728319	165.33070
226.	3.4899034	164.56022
227.	3.8443713	164.85080
228.	4.3693820	166.24802
229.	5.2322027	169.77120
230.	1.0182813	-165.60600
231.	.91611772	-164.13332
232.	.85362642	-163.50245
233.	.79976933	-162.89356
234.	.76826352	-162.55455
235.	.76900960	-162.77557
236.	.78258238	-163.13041
237.	.82125881	-163.88891
238.	.89298309	-165.12379
239.	.99155335	-166.58995
240.	1.1152134	-168.18580
241.	1.2724424	-169.94772
242.	1.4649441	-171.82287
243.	1.6935393	-173.77752
244.	1.9546477	-175.78511
245.	2.2464576	-177.84262
246.	2.5610418	-179.93537

247.	2.8795196	177.95661
248.	3.1896662	175.87351
249.	3.4633445	173.86999
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503.	.74386226	-25.487274
504.	.75620904	-25.663329

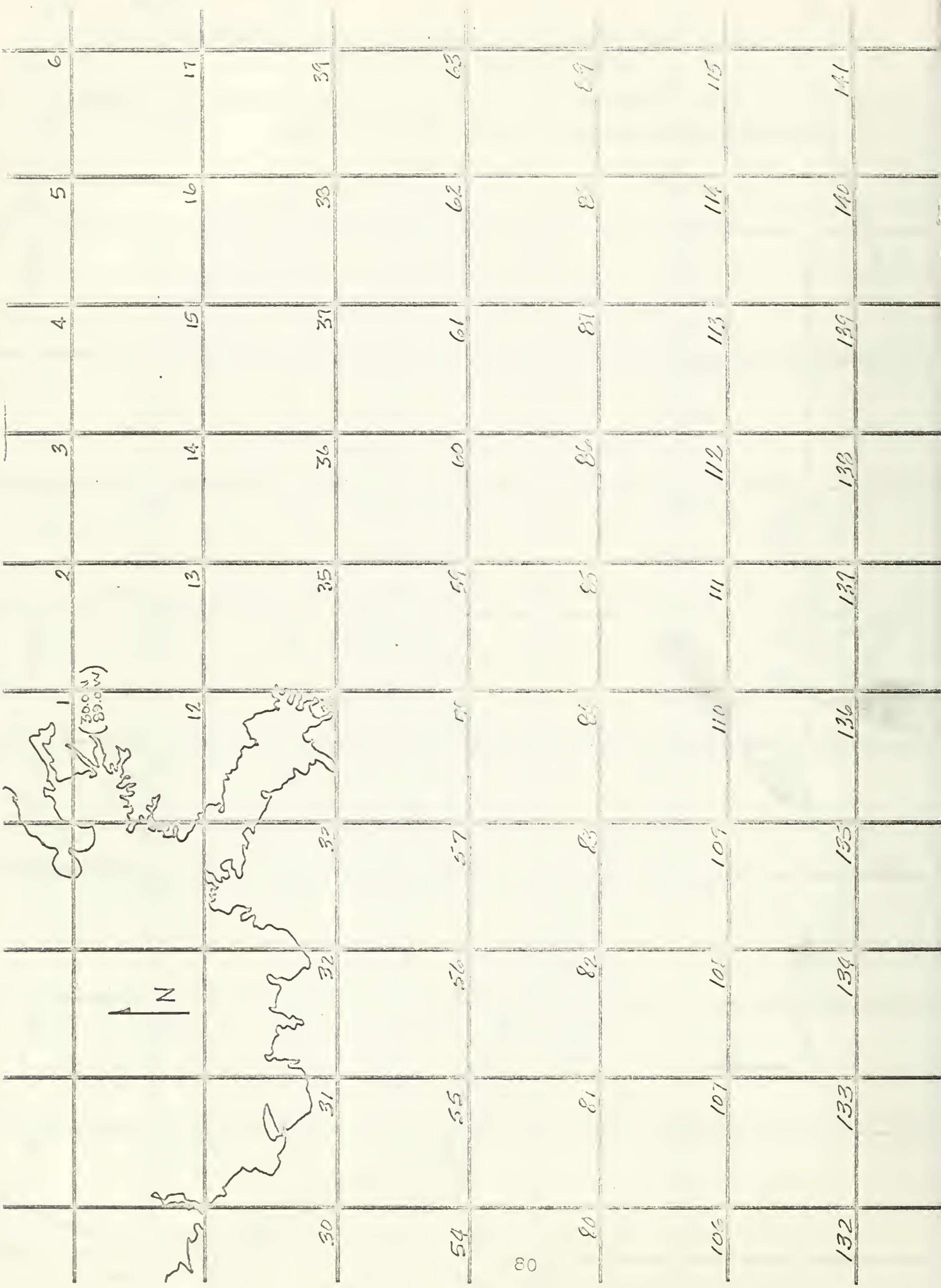
NUMERICAL RESULTS OF HAND CALCULATIONS

POINT	MAGNITUDE (cm)	PHASE (Degrees)
1.	1.414	315°
2.	9.86	24°
3.	10.1	354°
4.	1.414	45°
5.	7	0
6.	14	0
7.	3	0
8.	11.4	343.3°
9.	6	90°
10.	3.14	14°
11.	7.07	8.14°
12.	6.0	0
13.	4.0	0
14.	4.47	60°
15.	7.07	8.14°
16.	11.05	5.2°

NUMERICAL RESULTS OF 16 POINT GRID
COMPUTER SOLUTION

POINT	AMPLITUDE (cms.)	PHASE (DEGREES)
1.	3.47	27.8
2.	3.43	37.35
3.	3.16	45.19
4.	3.16	56.61
5.	4.60	13.02
6.	4.94	14.65
7.	4.95	17.13
8.	3.74	32.58
9.	4.18	49.20
10.	5.72	9.11
11.	6.37	8.11
12.	8.28	3.06
13.	3.04	2.98
14.	5.54	5.41
15.	6.49	7.76
16.	6.98	7.30

APPENDIX E - 50[illegible] POINT GRID ARRANGEMENT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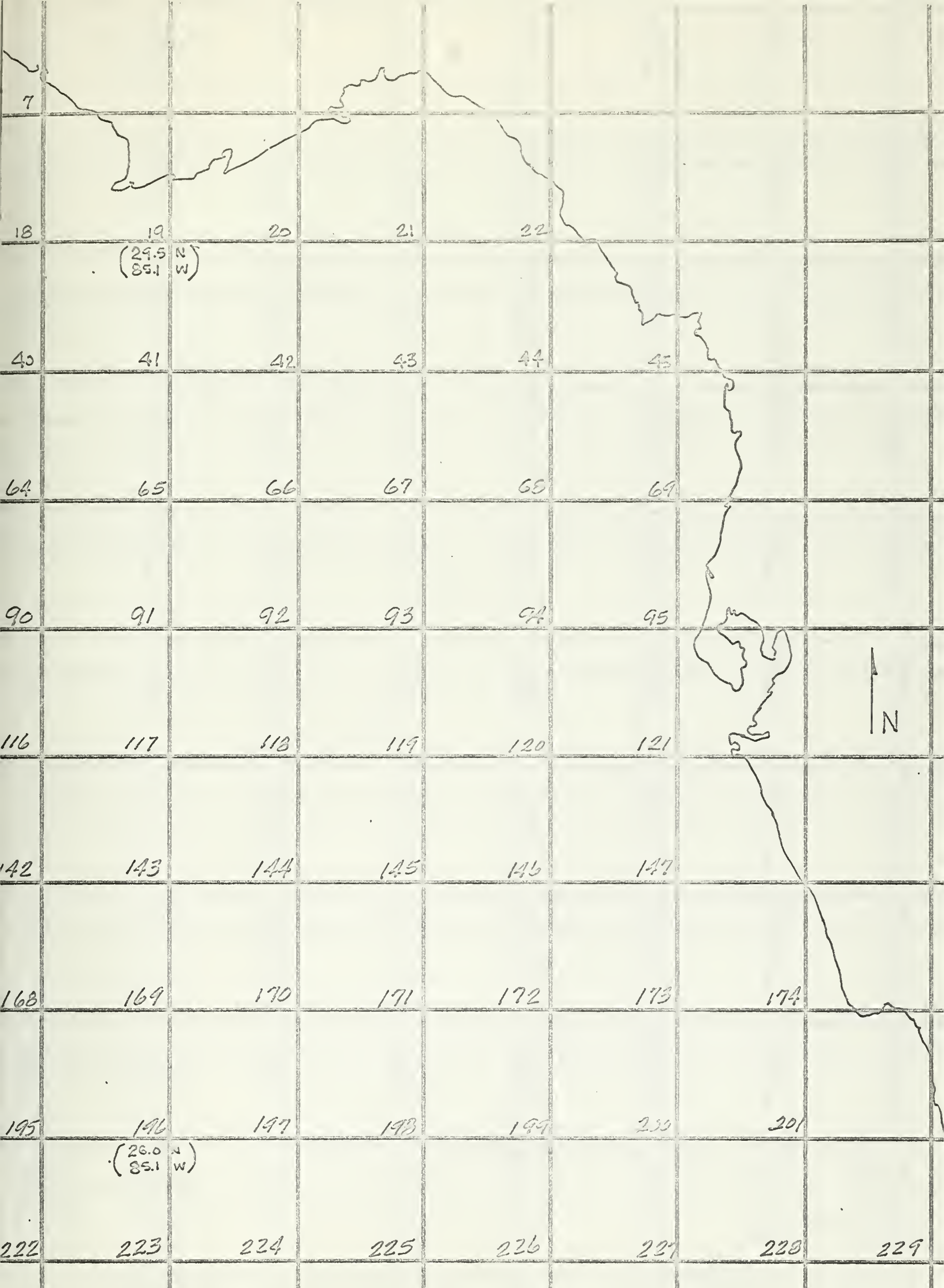
(89.0 W)
N


7
18
19
(29.5 N 85.1 W)
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201
(26.0 N 85.1 W)
222
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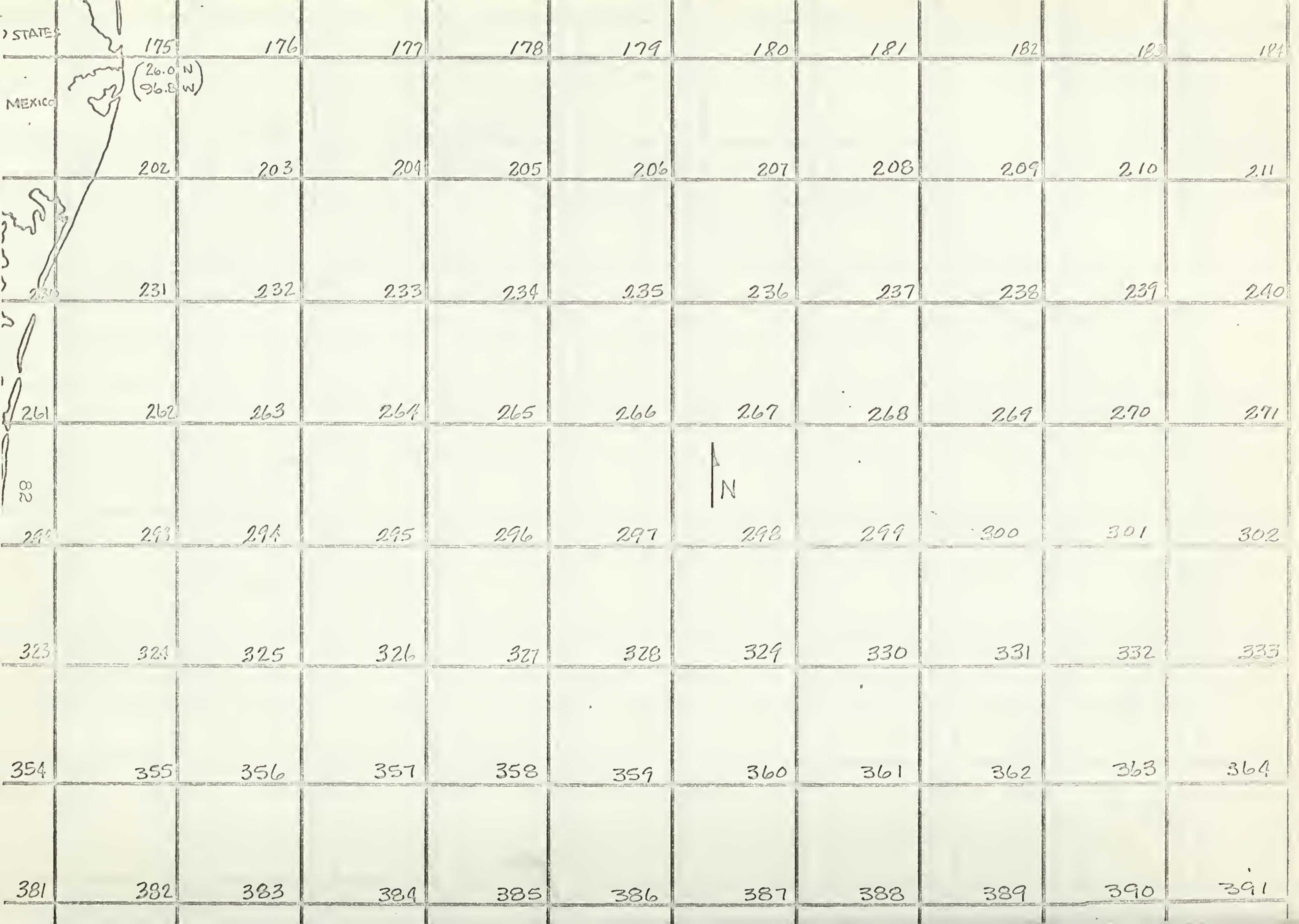
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N
175
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396
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368
395
187
214
243
274
305
336
367
394
186
213
242
273
304
335
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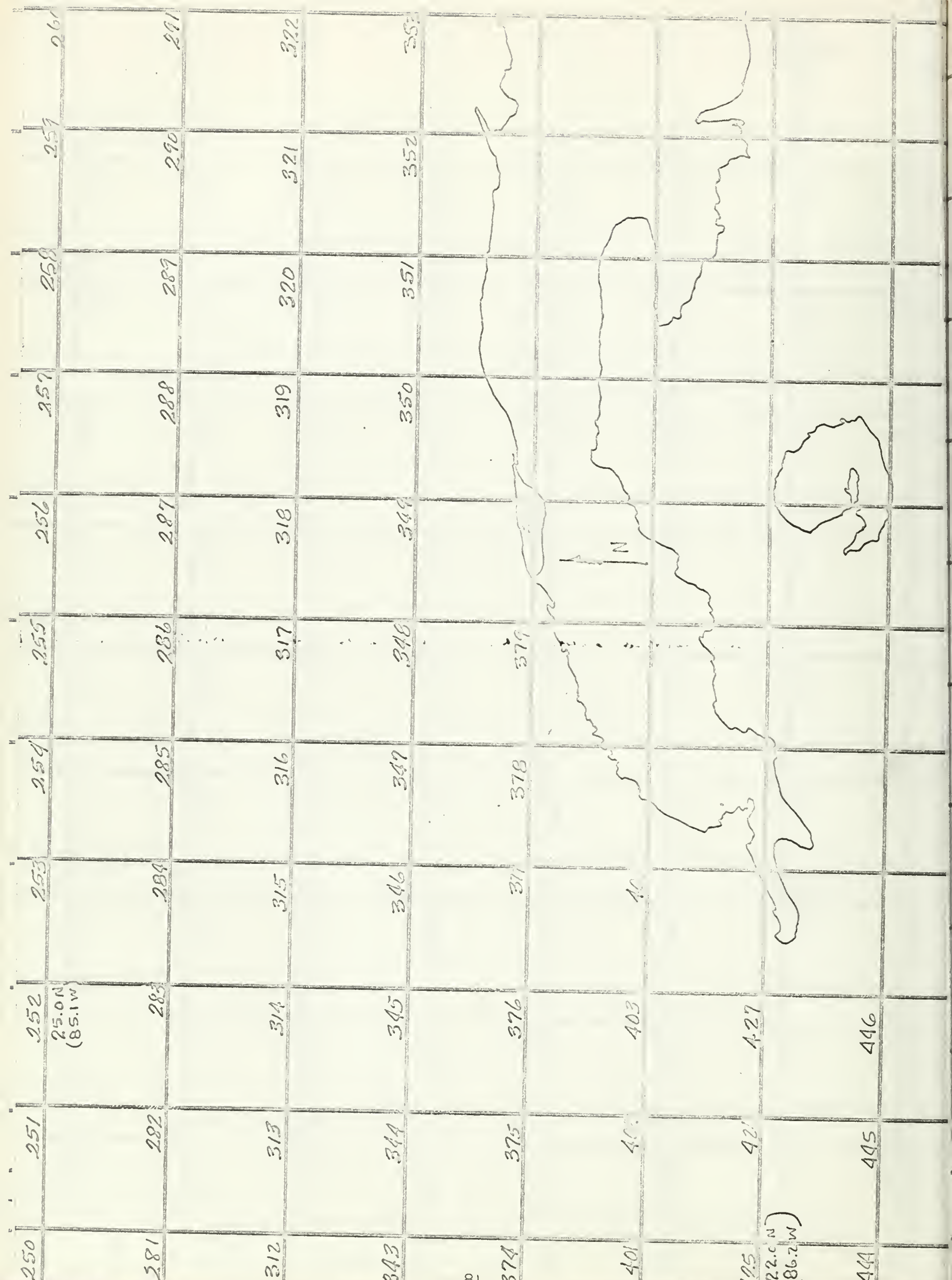
393
185
(26.0°N 91.2°W)
212
241
272
303
334
365
392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287
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401
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444
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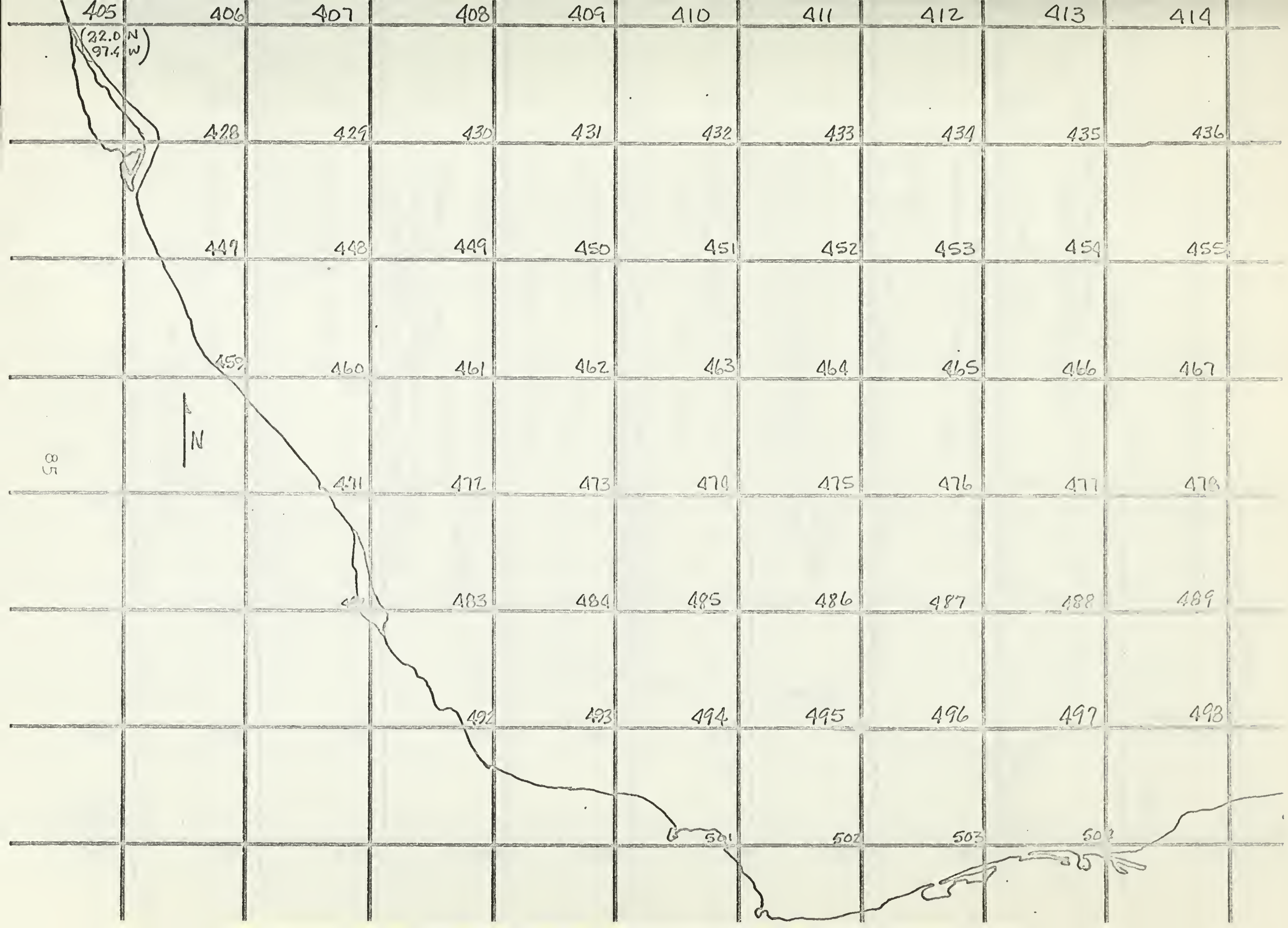
86.2W
N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414
(22.0 N
97.4 W)
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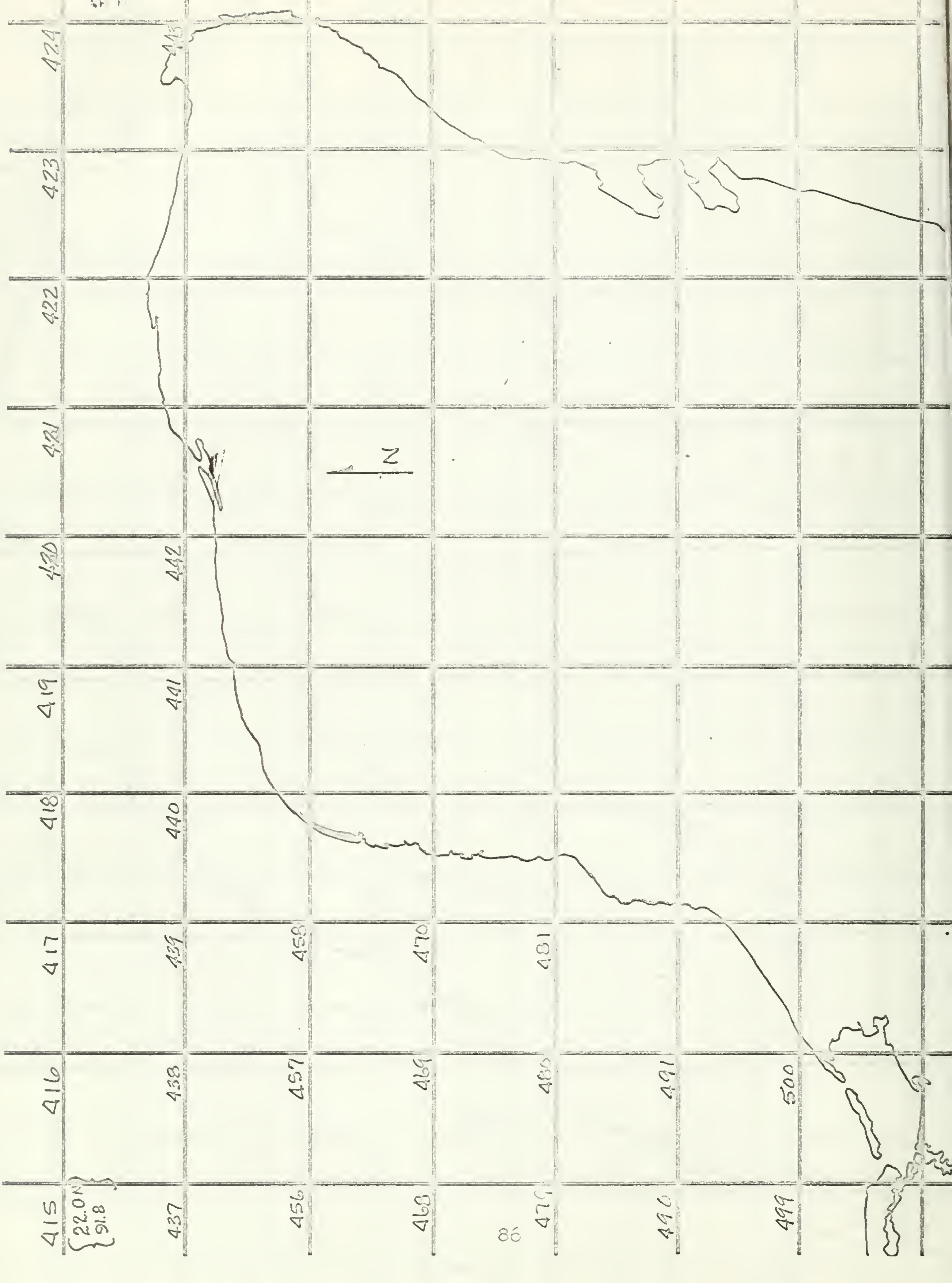
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457
458
468
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13. ABSTRACT

An ability to predict tides in the open sea is of value to oceanographers and coastal engineers. This study attempts to attain this goal of deep-sea tidal prediction by application of the hydrodynamic equations to individual tidal constituents. Similar treatments by Hansen and Rossiter with less general application were very success--ful. Adaptation of the method to the digital computer contributed considerably to its usefulness and versatility. Specific solutions are sought in the Gulf of Mexico for the M2 tidal constituent in an attempt to explain abrupt shift in tidal forms along the Gulf Coast. One hand calculation solution using relaxation techniques on a course grid was undertaken. Computer solutions were obtained for actual depth with a coarse grid and for constant depth with a much finer grid. Cotidal and corange analyses of the computer solutions both fit well with coastal data indicating that bathymetry is not as influential in the tidal form shifts as expected. The technique utilized is judged effective and its refinement and application to other basins is recommended.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
TIDES GULF OF MEXICO TIDAL PREDICTION M2 CONSTITUENT NUMERICAL PREDICTION OF TIDES						

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